CHAPTER 4

EXPONENTS AND POWERS

INTRODUCTION

Do you remember the power or exponent of integers?

$$4^2$$

2 is the **power** or **exponent**

4 is called the **base**

$$(-5)^3$$

3 is the **exponent**

– 5 is the **base**

In the same way, in

$$\left(\frac{4}{3}\right)^3, \quad \left(\frac{-3}{5}\right)^6 \quad \text{and} \quad \left(\frac{6}{7}\right)^4$$

$\frac{4}{3}$, $\frac{-3}{5}$ and $\frac{6}{7}$ are the bases and 3, 6 and 4 are the exponents respectively. Let us study more about numbers with the base as a rational number and the exponent as an integer.

EXPONENTS OF RATIONAL NUMBERS

We consider the following examples:

Example 1: Express the following in the exponential form and also write their respective base and exponent.

(i) $\frac{3}{8} \times \frac{3}{8} \times \frac{3}{8} \times \frac{3}{8} \times \frac{3}{8}$

(ii) $\left(-\frac{2}{3}\right) \times \left(-\frac{2}{3}\right) \times \left(-\frac{2}{3}\right)$

Solution: (i) $\frac{3}{8}\times\frac{3}{8}\times\frac{3}{8}\times\frac{3}{8}\times\frac{3}{8}=\left(\frac{3}{8}\right)^5$

Here, Base = $\frac{3}{8}$, Exponent = 5

(ii) $\left(-\frac{2}{3}\right)\times\left(-\frac{2}{3}\right)\times\left(-\frac{2}{3}\right)=\left(-\frac{2}{3}\right)^3$

Here, Base = $-\frac{2}{3}$, Exponent = 3

Example 2: Express the following as rational numbers.

(i) $\left(\frac{5}{7}\right)^3$ (ii) $\left(-\frac{3}{4}\right)^4$

Solution: (i) $\left(\frac{5}{7}\right)^3=\frac{5}{7}\times\frac{5}{7}\times\frac{5}{7}=\frac{5\times5\times5}{7\times7\times7}=\frac{5^3}{7^3}=\frac{125}{343}$

(ii) $\left(-\frac{3}{4}\right)^4=\frac{(-3)\times(-3)\times(-3)\times(-3)}{4\times4\times4\times4}$

$$=\frac{(-3)^4}{4^4}=\frac{81}{256}$$

From the above examples, we conclude that:

If $\frac{p}{q}$ is a rational number and m is any positive integer, then

$$\left(\frac{p}{q}\right)^m=\frac{p^m}{q^m}$$

Example 3: Express the following rational numbers in exponential form.

(i) $\frac{8}{125}$ (ii) $-\frac{144}{169}$

Solution: (i) Since 8 = 2 × 2 × 2 and 125 = 5 × 5 × 5,

(prime factorisation of 8 and 125)

we have, $\frac{8}{125}=\frac{2\times2\times2}{5\times5\times5}=\frac{2^3}{5^3}=\left(\frac{2}{5}\right)^3$

(ii) Since $-144 = -(12 \times 12)$ and $169 = 13 \times 13$,

we have, $$-\frac{144}{169} = \frac{-(12\times 12)}{13\times 13}$$

$$= \frac{-(12)^2}{13^2} = -\left(\frac{12}{13}\right)^2$$

Worksheet 1

1. Write the base and exponent in each of the following:

(i) $\left(-\frac{1}{3}\right)^3$ (ii) $\left(-\frac{4}{7}\right)^6$ (iii) $\left(\frac{2}{9}\right)^5$

(iv) $\left(\frac{15}{19}\right)^3$ (v) $(-15)^4$ (vi) $-\frac{2}{3}$

2. Express the following in exponential form.

(i) $\frac{5}{6}\times\frac{5}{6}$ (ii) $\frac{9}{2}\times\frac{9}{2}\times\frac{9}{2}\times\frac{9}{2}$

(iii) $\left(-\frac{7}{8}\right)\times\left(-\frac{7}{8}\right)\times\left(-\frac{7}{8}\right)$ (iv) $\left(-\frac{1}{2}\right)\times\left(-\frac{1}{2}\right)\times\left(-\frac{1}{2}\right)\times\left(-\frac{1}{2}\right)\times\left(-\frac{1}{2}\right)$

(v) $1.8 \times 1.8 \times 1.8 \times 1.8 \times 1.8 \times 1.8 \times 1.8$ (vi) $\frac{11}{12}\times\frac{11}{12}\times\frac{11}{12}\times\frac{11}{12}\times\frac{11}{12}\times\frac{11}{12}$

3. Express the following as rational numbers in the form $\frac{p}{q}$.

(i) $\left(\frac{5}{6}\right)^3$ (ii) $\left(-\frac{12}{13}\right)^2$ (iii) $\left(\frac{4}{9}\right)^4$

(iv) $\left(-\frac{1}{2}\right)^5$ (v) $\left(\frac{1}{4}\right)^4$ (vi) $\left(\frac{3}{5}\right)^3$

4. Express the following as powers of rational numbers.

(i) $\frac{81}{625}$ (ii) $-\frac{8}{125}$ (iii) $-\frac{343}{512}$

(iv) $\frac{32}{243}$ (v) $-\frac{1}{216}$ (vi) $\frac{729}{1000}$

RECIPROCALS WITH POSITIVE INTEGRAL EXPONENTS

We know that reciprocal of 3 is $\frac{1}{3}$, reciprocal of 3^2 is $\frac{1}{3^2}$. Let us do some examples to find the reciprocals of the rational numbers with positive integral exponents.

Example 4: Find the reciprocal of:

(i) $\left(\frac{3}{4}\right)^2$ (ii) $\left(-\frac{5}{7}\right)^5$

Solution: (i) Reciprocal of $\left(\frac{3}{4}\right)^2$

$$= \frac{1}{\left(\frac{3}{4}\right)^2}$$

$$= \frac{1}{\frac{3^2}{4^2}}$$

$$= \frac{4^2}{3^2} = \left(\frac{4}{3}\right)^2$$

(ii) Reciprocal of $\left(\frac{-5}{7}\right)^5$

$$= \frac{1}{\left(\frac{-5}{7}\right)^5} = \frac{1}{\frac{(-5)^5}{(7)^5}}$$

$$= \frac{7^5}{(-5)^5}$$

$$= \left(-\frac{7}{5}\right)^5$$

In general, the reciprocal of a rational number $\frac{p}{q}$ where $p \neq 0$; $q \neq 0$ is $\frac{q}{p}$.

The reciprocal of $\left(\frac{p}{q}\right)^m$ is $\left(\frac{q}{p}\right)^m$, where m is a positive integer.

Worksheet 2

1. Find the reciprocals of each of the following:

(i) $(12)^5$ (ii) $(-9)^4$ (iii) $\left(\frac{2}{7}\right)^3$

(iv) $\left(\frac{-3}{7}\right)^7$ (v) $\left(-\frac{1}{15}\right)^8$ (vi) $\left(-\frac{8}{13}\right)^{99}$

(vii) $\left(\frac{1}{10}\right)^{10}$ (viii) $\frac{2}{3} \times \frac{9}{4}$ (ix) $\left(\frac{1}{2}\right)^{2} \times 4$

(x) $-\left(\frac{7}{8}\right)^{2}$

LAWS OF EXPONENTS

1. We know that

$$2^3 = 2 \times 2 \times 2 \text{ and } 2^4 = 2 \times 2 \times 2 \times 2$$

Now, let us find the value of $2^3 \times 2^4$.

$$\begin{aligned} 2^3 \times 2^4 &= (2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2) \\ &= 2^7 \end{aligned}$$

In other words $2^3 \times 2^4 = 2^{3+4} = 2^7$

In the above example, we learn that when we multiply two numbers having the same base, we simply add the two exponents.

Example 5: Find the product of $\left(\frac{2}{5}\right)^{2} \times \left(\frac{2}{5}\right)^{3}$.

Solution: $\left(\frac{2}{5}\right)^{2} \times \left(\frac{2}{5}\right)^{3} = \left(\frac{2}{5}\right)^{2+3} = \left(\frac{2}{5}\right)^{5}$

Law I: **If x is any rational number ($x \neq 0$), and m and n are positive integers then, $x^m \times x^n = x^{m+n}$**

Remember

Law I can also be extended when the numbers in the exponential form to be multiplied are more than two but each has the same base.

For example,

$$\left(\frac{2}{3}\right)^{2} \times \left(\frac{2}{3}\right)^{3} \times \left(\frac{2}{3}\right)$$

$$= \left(\frac{2}{3}\right)^{2+3+1} = \left(\frac{2}{3}\right)^{6}$$

2. (a) Simplify $3^5 \div 3^2$

$$3^5 \div 3^2 = \frac{3^5}{3^2}$$

$$= \frac{3 \times 3 \times 3 \times 3 \times 3}{3 \times 3}$$

$$= 3 \times 3 \times 3 = 3^3$$

or $$\frac{3^5}{3^2} = 3^{5-2}$$

$$= 3^3$$

Here, the dividend and the divisor have the same base. Therefore, during the process of division, the power of divisor is subtracted from the power of dividend.

Example 6: Simplify $\left(\frac{2}{3}\right)^6 \div \left(\frac{2}{3}\right)^4$ and express the result in exponential form.

Solution:

$$\left(\frac{2}{3}\right)^6 \div \left(\frac{2}{3}\right)^4 = \frac{\left(\frac{2}{3}\right)^6}{\left(\frac{2}{3}\right)^4}$$

$$= \frac{\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}}{\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}}$$

$$= \frac{2}{3} \times \frac{2}{3} = \left(\frac{2}{3}\right)^2$$

or $$\left(\frac{2}{3}\right)^6 \div \left(\frac{2}{3}\right)^4 = \left(\frac{2}{3}\right)^{6-4} = \left(\frac{2}{3}\right)^2$$

Law II (a): If x be any rational number ($x \neq 0$), m and n be positive integers such that $m > n$ then, $x^m \div x^n = x^{m-n}$

Worksheet 3

1. Which of the following statements are true?

(i) $\left(+\frac{3}{4}\right)^3 \times \left(-\frac{3}{4}\right)^3 = \left(-\frac{3}{4}\right)^6$

(ii) $\left(\frac{4}{7}\right)^5 \times \left(\frac{4}{7}\right)^3 = \left(\frac{4}{7}\right)^8$

(iii) $\left(-\frac{1}{2}\right)^4 \div \left(-\frac{1}{2}\right)^3 = \left(-\frac{1}{2}\right)$ (iv) $\left(\frac{6}{7}\right)^6 \div \left(\frac{6}{7}\right)^0 = \left(\frac{6}{7}\right)^0$

2. Fill in the blank in each of the following so as to make the statement true.

(i) $\left(-\frac{2}{3}\right) \times \left(-\frac{2}{3}\right)^3 = \left(-\frac{2}{3}\right)^{\_\_\_}$ (ii) $\left(\frac{4}{5}\right)^{\_\_\_} \times \left(\frac{4}{5}\right)^9 = \left(\frac{4}{5}\right)^{11}$

(iii) $\left(-\frac{3}{7}\right)^{15} \div \left(-\frac{3}{7}\right)^{\_\_\_} = \left(-\frac{3}{7}\right)$ (iv) $\left(-\frac{1}{10}\right)^{\_\_\_} \div \left(-\frac{1}{10}\right)^8 = \left(-\frac{1}{10}\right)^8$

(v) $\left(\frac{2}{9}\right)^6 \div \left(\frac{2}{9}\right)^0 = \left(\frac{2}{9}\right)^{\_\_\_}$ (vi) $\left(\frac{-12}{13}\right)^2 \times \left(\frac{-12}{13}\right)^{\_\_\_} = \left(\frac{-12}{13}\right)^5$

3. Simplify and express the result in exponential form.

(i) $\left(-\frac{3}{7}\right)^3 \times \left(-\frac{3}{7}\right)^4$ (ii) $\left(\frac{11}{12}\right)^{15} \times \left(\frac{11}{12}\right)^5 \times \left(\frac{11}{12}\right)^{10}$

(iii) $\left(-\frac{9}{11}\right)^9 \div \left(-\frac{9}{11}\right)^7$ (iv) $\left(\frac{1}{4}\right)^8 \div \left(\frac{1}{4}\right)^6$

4. Simplify and express the result as a rational number.

(i) $\left(\frac{5}{7}\right)^4 \div \left(\frac{5}{7}\right)^2$ (ii) $\left(-\frac{2}{3}\right)^2 \times \left(-\frac{2}{3}\right)^3$ (iii) $\left(\frac{3}{4}\right)^3 \times \left(\frac{3}{4}\right)^2$

(iv) $\left(-\frac{3}{5}\right)^6 \div \left(-\frac{3}{5}\right)^3$ (v) $\left(-\frac{1}{10}\right)^4 \times \left(-\frac{1}{10}\right)^2$

5. Evaluate:

(i) $\left(\frac{3}{4}\right)^3 \times \left(\frac{2}{3}\right)^2$ (ii) $\left[\left(\frac{1}{3}\right)^6 \div \left(\frac{1}{3}\right)^5\right] \div \frac{1}{3}$

(iii) $(2^4 \times 2^5) \div 2^8$ (iv) $\left[4^2 - 3^2\right] \div \left(\frac{1}{7}\right)^2$

2. (b) Simplify $2^2 \div 2^5$

$$2^2 \div 2^5 = \frac{2^2}{2^5}$$

$$= \frac{2 \times 2}{2 \times 2 \times 2 \times 2 \times 2}$$

$$= \frac{1}{2 \times 2 \times 2}$$

$$= \frac{1}{2^3}$$

or $$2^2 \div 2^5 = \frac{1}{2^{5-2}}$$

Example 7: Simplify $\left(\frac{3}{7}\right)^4 \div \left(\frac{3}{7}\right)^6$ and express the result in exponential form.

Solution: $$\left(\frac{3}{7}\right)^4 \div \left(\frac{3}{7}\right)^6 = \frac{\left(\frac{3}{7}\right)^4}{\left(\frac{3}{7}\right)^6}$$

$$= \frac{\frac{3}{7} \times \frac{3}{7} \times \frac{3}{7} \times \frac{3}{7}}{\frac{3}{7} \times \frac{3}{7} \times \frac{3}{7} \times \frac{3}{7} \times \frac{3}{7} \times \frac{3}{7}}$$

$$= \frac{1}{\frac{3}{7} \times \frac{3}{7}} = \frac{1}{\left(\frac{3}{7}\right)^2}$$

or $$\left(\frac{3}{7}\right)^4 \div \left(\frac{3}{7}\right)^6 = \frac{1}{\left(\frac{3}{7}\right)^{6-4}} = \frac{1}{\left(\frac{3}{7}\right)^2}$$

In this case, the power of dividend is less than the power of divisor. So, the answer has to be written in reciprocal form.

The above examples suggest the following law:

Law II (b): **If x be any rational number ($x \neq 0$), m and n be positive integers such that m < n then $x^m \div x^n = \frac{1}{x^{n-m}}$**

3. Observe the following examples carefully:

(i) $$2^3 \div 2^3 = \frac{2 \times 2 \times 2}{2 \times 2 \times 2} = 1$$

we have, $$2^3 \div 2^3 = 2^{3-3} = 2^0 = 1$$

(ii) $$(-3)^2 \div (-3)^2 = \frac{(-3)\times(-3)}{(-3)\times(-3)} = 1$$

we have, $$(-3)^2 \div (-3)^2 = (-3)^{2-2} = (-3)^0 = 1$$

(iii) $$\left(-\frac{4}{9}\right)^4 \div \left(-\frac{4}{9}\right)^4 = \frac{\left(-\frac{4}{9}\right)\times\left(-\frac{4}{9}\right)\times\left(-\frac{4}{9}\right)\times\left(-\frac{4}{9}\right)}{\left(-\frac{4}{9}\right)\times\left(-\frac{4}{9}\right)\times\left(-\frac{4}{9}\right)\times\left(-\frac{4}{9}\right)}$$

$$= \left(-\frac{4}{9}\right)^{4-4} = \left(-\frac{4}{9}\right)^0 = 1$$

The above examples suggest us the following law:

Law III: If x is non-zero rational number, then x° = 1

4. Simplify $(2^3)^2$

$$(2^3)^2 = 2^3 \times 2^3$$
$$= (2 \times 2 \times 2) \times (2 \times 2 \times 2)$$
$$= 2^6$$

or $$(2^3)^2 = 2^{3 \times 2}$$

In the above example, we get the value of $(2^3)^2$ as 2^6.

Example 8: Simplify $\left[\left(-\frac{1}{3}\right)^2\right]^3$ and express the results in exponential form.

Solution:
$$\left[\left(-\frac{1}{3}\right)^2\right]^3 = \left(-\frac{1}{3}\right)^2 \times \left(-\frac{1}{3}\right)^2 \times \left(-\frac{1}{3}\right)^2$$
$$= \left(-\frac{1}{3}\right)\times\left(-\frac{1}{3}\right)\times\left(-\frac{1}{3}\right)\times\left(-\frac{1}{3}\right)\times\left(-\frac{1}{3}\right)\times\left(-\frac{1}{3}\right)$$
$$= \left(-\frac{1}{3}\right)^6 = \left(-\frac{1}{3}\right)^{2\times 3}$$

From the above examples, we conclude another law which states:

Law IV: If x be any rational number, x ≠ 0 and m, n be any integer,
then, $(x^m)^n = x^{m \times n} = x^{mn}$

Example 9: Find the value of x so that $\left[\left(\frac{2}{5}\right)^3\right]^2 = \left(\frac{2}{5}\right)^{2x}$

Solution:

$$\left[\left(\frac{2}{5}\right)^3\right]^2 = \left(\frac{2}{5}\right)^{2x}$$

$$\Rightarrow \quad \left(\frac{2}{5}\right)^{3\times 2} = \left(\frac{2}{5}\right)^{2x}$$

$$\Rightarrow \quad \left(\frac{2}{5}\right)^{6} = \left(\frac{2}{5}\right)^{2x}$$

Since bases are same, their exponents must be equal.

$$\therefore \quad 6 = 2x$$

$$\Rightarrow \quad x = \frac{6}{2}$$

$$\Rightarrow \quad x = 3$$

Worksheet 4

1. Fill in the blanks.

(i) $16^{15} \div 16^{19} = \frac{1}{16^{\_\_\_}}$

(ii) $\left(\frac{11}{12}\right)^{\_\_\_} \div \left(\frac{11}{12}\right)^{22} = \frac{1}{\left(\frac{11}{22}\right)^2}$

(iii) $\left(\frac{1}{3^2}\right)^{\_\_\_} = \frac{1}{3^6}$

(iv) $\left(\left(-\frac{1}{2}\right)^4\right)^2 = \left(-\frac{1}{2}\right)^{\_\_\_}$

(v) $\left(\frac{1}{12}\right)^5 \div \left(\frac{1}{12}\right)^5 = \left(\frac{1}{12}\right)^{\_\_\_}$

(vi) $\left[\left(-\frac{1}{7}\right)^0\right]^7 = \left(-\frac{1}{7}\right)^{\_\_\_}$

2. Simplify and express the result in exponential form.

(i) $\left[\left(\frac{9}{2}\right)^0\right]^5$

(ii) $\left[\left(-\frac{5}{17}\right)^6\right]^3$

(iii) $\left[\left(\frac{4}{7}\right)^4\right]^4$

(iv) $\left[\left(-\frac{2}{5}\right)^3 \times \left(-\frac{2}{5}\right)^2\right]^4$

(v) $\left(\frac{3}{5}\right)^3 \div \left(\frac{3}{5}\right)^8$

(vi) $\left[\left(\frac{12}{5}\right)^0 \times \left(\frac{12}{5}\right)\right]^5$

3. Evaluate:

(i) $\left(-\frac{3}{4}\right)^3 \div \left(-\frac{3}{4}\right)^5$ (ii) $\left(\frac{1}{5^2}\right)^2 \times \frac{1}{5}$ (iii) $\left[\left(-\frac{5}{6}\right)^2\right]^2 \div \left(-\frac{5}{6}\right)^2$

(iv) $\left(\frac{2}{3}\right)^2 \div \left[\left(\frac{2}{3}\right)^2\right]^0$ (v) $\frac{\left(\frac{1}{2}\right)^5}{\left(\frac{1}{2}\right)^3} - \frac{\left(\frac{1}{2}\right)^6}{\left(\frac{1}{2}\right)^5}$

4. Find the value of:

(i) $\left(\frac{5}{7}\right)^{3\times 2-6}$ (ii) $(3^\circ + 4^\circ) \times 5^\circ$

(iii) $1^\circ \times 2^\circ + 3^\circ \times 4^\circ + 5^\circ \times 6^\circ$ (iv) $\left(-\frac{5}{9}\right)^{9-3\times 2-3}$

5. Find the value of x so that–

(i) $\left(\frac{3}{4}\right)^{2x+1} = \left[\left(\frac{3}{4}\right)^3\right]^3$ (ii) $\left(\frac{2}{5}\right)^3 \times \left(\frac{2}{5}\right)^6 = \left(\frac{2}{5}\right)^{3x}$

(iii) $\left(-\frac{1}{5}\right)^{20} \div \left(-\frac{1}{5}\right)^{15} = \left(-\frac{1}{5}\right)^{5x}$ (iv) $\frac{1}{16} \times \left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^{3(x-2)}$

6. Which of the following statements are true?

(i) $(0.6)^8 \div (0.6)^7 = (0.6)^2$ (ii) $\left(\frac{12}{13}\right)^6 \div \left(\frac{12}{13}\right)^3 = \left(\frac{12}{13}\right)^3$

(iii) The reciprocal of $\left(\frac{7}{5}\right)^{12}$ is $\left(\frac{5}{7}\right)^{12}$ (iv) $(5 + 5)^5 = 5^5 + 5^5$

(v) $\left[\left(\frac{1}{4}\right)^4 \div \left(\frac{1}{4}\right)^3\right] \div \left(\frac{1}{4}\right) = \frac{1}{4}$ (vi) $\left(\frac{1}{7} \times \frac{1}{7^2}\right) \div \frac{1}{7^3} = 1$

5. Simplify $2^3 \div 2^5$

$$2^3 \div 2^5 = \frac{2^3}{2^5} = \frac{2\times 2\times 2}{2\times 2\times 2\times 2\times 2} = \frac{1}{2\times 2} = \frac{1}{2^2}$$

$$\frac{1}{2^2} = 2^{-2}$$

Or

$$2^3 \div 2^5 = 2^{3-5} = 2^{-2}$$

Example 10: Simplify $\left(\frac{2}{3}\right)^2 \div \left(\frac{2}{3}\right)^4$ and express the result in exponential form.

Solution:

$$\left(\frac{2}{3}\right)^2 \div \left(\frac{2}{3}\right)^4 = \frac{\left(\frac{2}{3}\right)^2}{\left(\frac{2}{3}\right)^4}$$

$$= \frac{\frac{2}{3} \times \frac{2}{3}}{\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}} = \frac{1}{\frac{2}{3} \times \frac{2}{3}}$$

$$= \frac{1}{\left(\frac{2}{3}\right)^2} = \left(\frac{2}{3}\right)^{-2}$$

Or

$$\left(\frac{2}{3}\right)^2 \div \left(\frac{2}{3}\right)^4 = \left(\frac{2}{3}\right)^{2-4}$$

$$= \left(\frac{2}{3}\right)^{-2}$$

From the above examples we conclude another law which states:

Law V: **If x be any rational number ($x \neq 0$) and m be positive integer, then,**

$x^{-m} = \frac{1}{x^m}$, i.e. x^{-m} is the reciprocal of x^m

Note:

In the above law, put m = 1. We get $x^{-1} = \frac{1}{x}$.

$\Rightarrow$ x^{-1} is the reciprocal of x.

Put m = 2, we get

$$x^{-2} = \frac{1}{x^2}$$

$\Rightarrow$ x^{-2} is the reciprocal of x^2 and so on.

Example 11: Convert negative exponents to positive exponents in the following:

(i) $\left(\frac{3}{4}\right)^{-2}$ (ii) $\left(-\frac{2}{5}\right)^{-6}$ (iii) $\left(-\frac{3}{7}\right)^{-11}$

Solution:

(i) $\left(\frac{3}{4}\right)^{-2} = \frac{1}{\left(\frac{3}{4}\right)^2} = \frac{1}{\frac{3^2}{4^2}} = \frac{4^2}{3^2} = \left(\frac{4}{3}\right)^2$

(ii) $\left(-\frac{2}{5}\right)^{-6} = \dfrac{1}{\left(-\frac{2}{5}\right)^{6}} = \dfrac{1}{\frac{(-2)^6}{5^6}} = \dfrac{5^6}{(-2)^6} = \left(-\frac{5}{2}\right)^{6}$

(iii) $\left(-\frac{3}{7}\right)^{-11} = \dfrac{1}{\left(-\frac{3}{7}\right)^{11}} = \dfrac{1}{\frac{(-3)^{11}}{7^{11}}} = \dfrac{7^{11}}{(-3)^{11}} = \left(-\frac{7}{3}\right)^{11}$

The above example suggests the following result:

If $\frac{\mathbf{p}}{\mathbf{q}}$ be any non-zero rational number and m be any positive integer, then $\left(\frac{\mathbf{p}}{\mathbf{q}}\right)^{-\mathbf{m}} = \left(\frac{\mathbf{q}}{\mathbf{p}}\right)^{\mathbf{m}}$

Example 12: Simplify:

(i) $\left(\frac{2}{7}\right)^{-3} \div \left(\frac{2}{7}\right)^{-2}$

(ii) $\left(\frac{3}{5}\right)^{-4} \times \left(\frac{3}{5}\right)^{4} + \left(\frac{1}{4}\right)^{5} \times \left(\frac{1}{4}\right)^{-5}$

(iii) $\left(\frac{1}{2}\right)^{-3} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-1}$

(iv) $\left(\frac{-4}{5}\right)^{-2} \times \left(\frac{-4}{5}\right)^{3} \times \left[\left(\frac{-4}{5}\right)^{-1}\right]^{2}$

Solution: (i) $\left(\frac{2}{7}\right)^{-3} \div \left(\frac{2}{7}\right)^{-2} = \left(\frac{7}{2}\right)^{3} \div \left(\frac{7}{2}\right)^{2}$

$= \left(\frac{7}{2}\right)^{3-2}$

$= \left(\frac{7}{2}\right)^{1}$

$= \frac{7}{2}$

(ii) $\left(\frac{3}{5}\right)^{-4} \times \left(\frac{3}{5}\right)^{4} + \left(\frac{1}{4}\right)^{5} \times \left(\frac{1}{4}\right)^{-5}$

$= \left(\frac{3}{5}\right)^{-4+4} + \left(\frac{1}{4}\right)^{5+(-5)}$

$= \left(\frac{3}{5}\right)^{0} + \left(\frac{1}{4}\right)^{0}$

$= 1 + 1 = 2$

(iii) $\left(\frac{1}{2}\right)^{-3} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-1} = 2^3 + 3^2 + 4^1$

$= 8 + 9 + 4$

$= 21$

(iv) $\left(-\frac{4}{5}\right)^{-2} \times \left(-\frac{4}{5}\right)^{3} \times \left[\left(-\frac{4}{5}\right)^{-1}\right]^{2}$

$= \left(\frac{-4}{5}\right)^{-2+3} \times \left(\frac{-4}{5}\right)^{-1\times 2}$

$= \left(-\frac{4}{5}\right)^{1} \times \left(-\frac{4}{5}\right)^{-2}$

$= \left(-\frac{4}{5}\right)^{1+(-2)}$

$= \left(-\frac{4}{5}\right)^{-1}$

$= -\frac{5}{4}$

Example 13: By what number should $(-3)^{-2}$ be multiplied so that the product may be equal to 9?

Solution: Let $(-3)^{-2}$ be multiplied by x to get 9. Then,

$(-3)^{-2} \times x = 9 \Rightarrow (-3)^{-2} \times x = (-3)^2$

$x = (-3)^2 \div (-3)^{-2}$

$= (-3)^2 \div \frac{1}{(-3)^2}$

$= (-3)^2 \times (-3)^2$

$= (-3)^4 = 81.$

Example 14: By what number should $(-12)^{-1}$ be divided so that the quotient may be equal to $(-4)^{-1}$?

Solution: Let $(-12)^{-1}$ be divided by x to get $(-4)^{-1}$. Then,

$(-12)^{-1} \div x = (-4)^{-1}$

$\Rightarrow \quad \frac{1}{-12} \times \frac{1}{x} = \frac{1}{-4} \quad \Rightarrow \quad \frac{1}{x} = \frac{1}{-4} \times \frac{-12}{1}$

$$\Rightarrow \qquad \frac{1}{x} = \frac{-12}{-4} = 3$$

$$1 = 3x$$

$$x = \frac{1}{3}$$

Example 15: If $\frac{p}{q} = \left(\frac{2}{3}\right)^{-3} \div \left(\frac{3}{4}\right)^{0}$, find $\left(\frac{p}{q}\right)^{-2}$

Solution: From the above statement,

$$\frac{p}{q} = \left(\frac{3}{2}\right)^{3} \div 1 \qquad \left[\because \left(\frac{3}{4}\right)^{0} = 1\right]$$

$$= \left(\frac{3}{2}\right)^{3} = \frac{3^3}{2^3} = \frac{27}{8}$$

Therefore, $$\left(\frac{p}{q}\right)^{-2} = \left(\frac{27}{8}\right)^{-2}$$

$$= \left(\frac{8}{27}\right)^{2} = \frac{8^2}{27^2} = \frac{64}{729}$$

Hence, $$\left(\frac{p}{q}\right)^{-2} = \frac{64}{729}$$

Worksheet 5

1. Simplify:

(i) $(2^{-1} - 3^{-1})^{-1} + (6^{-1} - 8^{-1})^{-1}$ (ii) $(2^{-1} \times 3^{-1})^{2} \times \left(-\frac{3}{8}\right)^{-1}$ (iii) $(4^{-1} \times 3^{-1}) \div 12^{-1}$

(iv) $\left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2}$ (v) $(2^{-1} \div 5^{-1})^{2} \times \left(\frac{-5}{8}\right)^{-1}$ (vi) $\left(\frac{3}{7}\right)^{-2} \div \left(\frac{4}{7}\right)^{-2}$

(vii) $\left[\left(\frac{2}{5}\right)^{-1} \times \left(\frac{3}{4}\right)^{-1}\right]^{-1}$ (viii) $(4^{5} \div 4^{8}) \times 64$ (ix) $\left(\frac{4}{5}\right)^{-3} \times \left(\frac{4}{5}\right)^{2} \times \left(\frac{4}{5}\right)^{3}$

(x) $\dfrac{\left(-\frac{1}{2}\right)^{-3}}{\left(-\frac{1}{2}\right)^{-4}} - \dfrac{\left(-\frac{1}{2}\right)^{-5}}{\left(-\frac{1}{2}\right)^{-6}}$

2. By what number should we multiply (2^{-5}) so that the product may be equal to (2^{-1})?

3. By what number should $\left(-\frac{1}{5}\right)^{-1}$ be multiplied so that the product may be equal to $\left(\frac{1}{5}\right)^{-3}$?

4. By what number should $(24)^{-1}$ be divided so that the quotient may be equal to $(4)^{-1}$?

5. By what number should $\left(-\frac{2}{3}\right)^{3}$ be divided so that the quotient may be equal to $\left(\frac{9}{4}\right)^{-2}$?

6. Find the value of x so that—

(i) $\left(\frac{3}{4}\right)^{-9} \times \left(\frac{3}{4}\right)^{-7} = \left(\frac{3}{4}\right)^{4x}$

(ii) $\left(\frac{2}{9}\right)^{-6} \times \left(\frac{2}{9}\right)^{3} = \left(\frac{2}{9}\right)^{2x-1}$

(iii) $\left(\frac{5}{7}\right)^{2} \div \left(\frac{5}{7}\right)^{3x+1} = \left(\frac{5}{7}\right)^{4}$

(iv) $\left(\frac{-6}{11}\right)^{x} \div \left[\left(\frac{-6}{11}\right)^{-2}\right]^{-1} = \left[\left(\frac{-6}{11}\right)^{2}\right]^{-3}$

7. If $\frac{p}{q} = \left(\frac{2}{3}\right)^{2} \times \left(\frac{1}{3}\right)^{-4}$, find the value of $\left(\frac{p}{q}\right)^{-2}$.

8. If $a = \left(\frac{3}{5}\right)^{-2} \div \left(\frac{7}{5}\right)^{0}$, find the value of a^{-3}.

9. Simplify:

$$\left[\left(\frac{2}{3}\right)^{2}\right]^{3} \times \left(\frac{2}{3}\right)^{-4} \times 3^{-1} \times \frac{1}{6}$$

10. Find the reciprocals of:

(i) $\left(\frac{1}{2}\right)^{-2} \div \left(\frac{2}{3}\right)^{-3}$

(ii) $\left(\frac{2}{5}\right)^{3} \times \left(\frac{5}{4}\right)^{2}$

11. Express the following as a rational number with positive exponent.

(i) $\left(\frac{3}{2}\right)^{-4}$

(ii) $(3^{-3})^{2}$

(iii) $7^{2} \times 7^{-3}$

(iv) $\left(\left(\frac{5}{8}\right)^{-2}\right)^{3}$

(v) $\left(\frac{2}{5}\right)^{-3} \times \left(\frac{2}{5}\right)^{5}$

(vi) $(8^{3} \div 8^{5}) \times 8^{-4}$

12. Express the following as a rational number with negative exponent.

(i) $\left(\frac{1}{7}\right)^5$ (ii) $(3^2)^9$ (iii) $5^3 \times 5^2$

(iv) $\left(\left(\frac{-8}{9}\right)^3\right)^2$ (v) $\left(\frac{5}{7}\right)^3 \div \left(\frac{5}{7}\right)^2$ (vi) $(2^6 \div 2^5) \times 2^2$

OTHER LAWS OF EXPONENTS

Following are two more laws of exponents, when the bases are different but the exponents are same.

Law VI: **If x and y are two rational numbers and m is any integer, then $(x \times y)^m = x^m \times y^m$**

Law VII: **If x and y are two rational numbers y ≠ 0 and m is any integer, then**

$$\left(\frac{x}{y}\right)^m = x^m \div y^m$$

Let us verify the above laws.

(i) Find out if $2^3 \times 3^3$ is equal to $(2 \times 3)^3$

L.H.S. $2^3 \times 3^3 = (2 \times 2 \times 2) \times (3 \times 3 \times 3)$

$= 8 \times 27$

$= 216$

R.H.S. $(2 \times 3)^3 = 6^3$

$= 6 \times 6 \times 6$

$= 216$

Thus, $2^3 \times 3^3 = (2 \times 3)^3$

(ii) Find out if $(-2)^4 \div (-3)^4$ is equal to $\left(\frac{-2}{-3}\right)^4$

L.H.S. $(-2)^4 \div (-3)^4 = [(-2) \times (-2) \times (-2) \times (-2)] \div [(-3) \times (-3) \times (-3) \times (-3)]$

$= 16 \div 81 = \frac{16}{81}$

R.H.S. $\left(\frac{-2}{-3}\right)^4 = \left(\frac{-2}{-3}\right) \times \left(\frac{-2}{-3}\right) \times \left(\frac{-2}{-3}\right) \times \left(\frac{-2}{-3}\right) = \frac{16}{81}$

Thus, $(-2)^4 \div (-3)^4 = \left(\frac{-2}{-3}\right)^4 = \left(\frac{2}{3}\right)^4$.

Let us apply these laws in some examples.

Example 16: Find the value of:

(i) $\left(\frac{2}{3}\right)^4 \times \left(\frac{3}{5}\right)^4$ (ii) $\left(\frac{4}{5}\right)^2 \div \left(\frac{6}{5}\right)^2$

Solution: (i) $\left(\frac{2}{3}\right)^4 \times \left(\frac{3}{5}\right)^4 = \left(\frac{2}{\cancel{3}} \times \frac{\cancel{3}}{5}\right)^4$

$$= \left(\frac{2}{5}\right)^4 = \frac{16}{625}$$

(ii) $\left(\frac{4}{5}\right)^2 \div \left(\frac{6}{5}\right)^2 = \left(\frac{4}{5} \div \frac{6}{5}\right)^2$

$$= \left(\frac{4}{5} \times \frac{5}{6}\right)^2$$

$$= \left(\frac{4}{6}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

Worksheet 6

1. Simplify:

(i) $4^4 \times 5^{-4}$ (ii) $2^2 \times \left(-\frac{1}{3}\right)^2$ (iii) $\left(-\frac{2}{3}\right)^3 \times \left(-\frac{3}{5}\right)^3$

(iv) $\left(\frac{1}{2}\right)^{-2} \times \left(\frac{2}{5}\right)^{-2}$ (v) $\left(\frac{-5}{6}\right)^4 \div \left(\frac{-7}{6}\right)^4$ (vi) $\left(\frac{-2}{3}\right)^{-5} \times \left(\frac{-3}{2}\right)^{-5}$

2. Find the value of x so that–

(i) $3^2 \times (-4)^2 = (-12)^{2x}$ (ii) $\left(\frac{-3}{2}\right)^6 \times \left(\frac{4}{9}\right)^3 = \left(\frac{1}{2}\right)^{3x}$ (iii) $\left(\frac{4}{5}\right)^{-2} \div \left(\frac{-4}{5}\right)^{-2} = (1)^{3x}$

(iv) $\left(\frac{9}{4}\right)^3 \times \left(\frac{8}{9}\right)^3 = 2^{6x}$ (v) $\left(\frac{15}{4}\right)^3 \div \left(\frac{5}{4}\right)^3 = 3^x$

USE OF EXPONENTS IN EXPRESSING LARGE AND SMALL NUMBERS

Look at the numbers written below:

170,000,000 5,000,000,000 659,000,000,000

All these numbers are too large. In order to make them managable, we can write these using exponents.

5,000,000,000 can be written as

5×10^9

or 50×10^8

or 500×10^7, etc.

Thus, every large number can be expressed as $k \times 10^n$ where k is some number and n is a positive integer.

To write 5,000,000,000 in this form, we may take–

	$k = 5$	and	$n = 9$
or	$k = 50$	and	$n = 8$
or	$k = 500$	and	$n = 7$, etc.

But, the first form, where $1 \leq k < 10$ is called the **standard form.**

From the above example, we conclude that a given number can be written in the form $k \times 10^n$, where n is some integer and k is a terminating decimal lying between 1 and 10, i.e. $1 \leq k < 10$.

The examples given above show that the exponential notation helps us in writing large numbers. In the same way, we can write small numbers also with the help of exponential notation.

Let us take an example to illustrate the point. To write 0.000021 in the form $k \times 10^n$, where $1 \leq k < 10$ we can write

$$0.000021 = \frac{21}{1000000} = 21 \times 10^{-6} = 2.1 \times 10^{-5}$$

Example 17: Write the following numbers in the form $k \times 10^n$ where $1 \leq k < 10$ and n is an integer.

(i) 92340000 (ii) 0.000255

Solution: (i) $9\underbrace{\uparrow 2340000}_{\text{7 places}} = 9.234 \times 10^7$.

[Decimal point has to be moved 7 places from right to left. When it moves from right to left, the exponent of 10 is positive].

(ii) $0\underbrace{.\,0002}_{\text{4 places}}\uparrow 55 = 2.55 \times 10^{-4}$.

[Decimal point has to be moved 4 places from left to right. When it moves from left to right, the exponent of 10 is negative].

Example 18: Express the product of 2.5×10^5 and 2.1×10^{-3} in the form $k \times 10^n$.

Solution:
$$(2.5 \times 10^5) \times (2.1 \times 10^{-3}) = (2.5 \times 2.1) \times (10^5 \times 10^{-3})$$
$$= 5.25 \times 10^{5-3}$$
$$= 5.25 \times 10^2.$$

Thus, the product of (2.5×10^5) and (2.1×10^{-3}) in the form $k \times 10^n$ is 5.25×10^2.

Note that $k = 5.25$ is such that $1 \leq k < 10$.

Example 19: Express $\dfrac{1.2 \times 10^3}{2.4 \times 10^{-4}}$ in the form $k \times 10^n$.

Solution:
$$\frac{1.2 \times 10^3}{2.4 \times 10^{-4}} = \frac{12}{24} \times \frac{10^3}{10^{-4}}$$
$$= \frac{1}{2} \times 10^{3-(-4)}$$
$$= 0.5 \times 10^{3+4}$$
$$= 0.5 \times 10^7 = 5 \times 10^{-1} \times 10^7$$
$$= 5.0 \times 10^6.$$

Example 20: Write the following numbers in usual form.

(i) 1.235×10^5 (ii) 7.2×10^{-4}

Solution: (i)
$$1.235 \times 10^5 = 1.235 \times 100{,}000$$
$$= 123500$$

(ii)
$$7.2 \times 10^{-4} = 7.2 \times \frac{1}{10^4}$$
$$= \frac{7.2}{10000}$$
$$= 0.00072$$

Worksheet 7

1. Write each of the following numbers in the form $k \times 10^n$ where $1 \leq k < 10$ and n is an integer.

(i) 1,384,000 (ii) 12.32005 (iii) 2157.957 (iv) 0.00002

(v) 0.00729 (vi) 0.000000000926 (vii) 520,000,000 (viii) 0.0000085

2. **Write the following numbers in the usual form.**

(i) 52.5×10^4 (ii) 158.9×10^6 (iii) 9.545×10^{12} (iv) 1.72×10^{-5}

(v) 8.5×10^{-7} (vi) 2.9×10^{-9} (vii) 6293.2×10^5 (viii) 1.925×10^{-6}

3. **Express each of the following in the form $k \times 10^n$; $(1 \le k < 10)$**

(i) $(1.25 \times 10^7) \div (5 \times 10^3)$ (ii) $(2.5 \times 10^{10}) \times (31.25 \times 10^{-5})$

(iii) $(1.6 \times 10^9) \times (5.0 \times 10^{-3})$ (iv) $[(3.4 \times 10^4) \times (5.0 \times 10^{-3})] \div [2.0 \times 10^5]$

4. **Express the numbers appearing in the following statements in the form $k \times 10^n$ where $1 \le k < 10$ and n is an integer.**

(i) Sun's diameter is 1,384,000 km.

(ii) The distance of the sun from the earth is approximately 150,000,000 km.

(iii) The speed of the light is about 27600 km/sec.

(iv) 1 Angstrom unit = $\dfrac{1}{10{,}000{,}000{,}000}$ m.

VALUE BASED QUESTIONS

1. **Sneha wants to show gratitude towards her teacher by giving her a self-made card. She chose a yellow coloured rectangular sheet of paper. The length of the sheet is (2^5) cm and breadth is (2^4) cm.**

(a) Find the area of the paper she has to work for making the card.

(b) What value of Sneha is depicted here?

2. **Aditya wanted to donate some cloth material to some poor people in a locality. For this his mother purchased (4^3) m cloth and donated (2^2) m to each person.**

(a) How many poor people would have received the cloth material?

(b) What value of Aditya is depicted here?

BRAIN TEASERS

1. A. **Tick (✓) the correct option.**

(a) The exponential form of $\left[\left(2^2\right)^3\right]^2$ is–

(i) 4^5 (ii) 2^{12} (iii) $(12)^2$ (iv) 2^7

(b) 2.7×10^{-3} is equal to–

(i) 0.000027 (ii) 0.00027 (iii) 0.0027 (iv) 2.007

(c) 9×4^2 is same as–

(i) $(12)^2$ (ii) $(36)^2$ (iii) $(36)^3$ (iv) $(18)^4$

(d) 10×10^{11} is equal to–

(i) $(100)^4$ (ii) $(10)^{10}$ (iii) $(100)^{12}$ (iv) $(10)^{12}$

(e) If $\left(\frac{3}{5}\right)^{2x} = 1$, then x is equal to–

(i) 2 (ii) 0 (iii) 1 (iv) $\frac{1}{2}$

B. Answer the following questions.

(a) Simplify $\left[\left\{\left(\frac{4}{9}\right)^2\right\}^0\right]^5$.

(b) Write $(2^2)^3 \times 3^6$ in simplified exponential form.

(c) Find the value of x if $\left[\left(\frac{3}{7}\right)^3\right]^{-2} = \left(\frac{3}{7}\right)^{2x}$.

(d) Simplify $6^{-2} + \left(\frac{3}{2}\right)^{-2}$.

(e) Write in usual form $11.2 \times (10)^{-7}$

2. Write the base and exponent of the following numbers.

(i) $\left(\frac{1}{9}\right)^{-4}$ (ii) $\frac{7}{11}$ (iii) $\left[\left(\frac{-5}{6}\right)^0\right]^2$

(iv) $\left(\frac{ac}{b}\right)^3$ (v) $\left(\frac{-1}{2} \times \frac{1}{3}\right)^0$ (vi) $\left(\frac{-11}{12}\right)^4$

3. Express the following as a rational number.

(i) $\left[\left(\frac{-1}{2}\right)^{-3}\right]^2$ (ii) $\left(\frac{1}{3^2}\right)^{-1} \times 9^{-2}$ (iii) $\left(\frac{16}{25}\right)^{-1} \div \left(\frac{4}{5}\right)^{-3}$

(iv) $\left(\frac{2}{3}\right)^{-1} - \left(\frac{3}{2}\right)^{-2}$ (v) $\left[\left(\frac{5}{6}\right)^0 + \left(\frac{3}{4}\right)^0\right] \div \left(\frac{2}{3}\right)^0$ (vi) $\dfrac{\left(\frac{-3}{5}\right)^7}{\left(\frac{-3}{5}\right)^7}$

4. **Express the following in the exponential form.**

(i) $(2.5)^4 \div \left(\frac{1}{2.5}\right)^2$ (ii) $(5^2 \times 5^5) \div 5^{10}$

5. **Find the reciprocals of:**

(i) $\left(\frac{2}{3}\right)^{-2} \div \left(\frac{2}{3}\right)^3$ (ii) $\left(\frac{3}{4}\right)^{-2} \times \left(\frac{3}{4}\right)^3$ (iii) $\left[\left(\frac{7}{8}\right)^{-3}\right]^2$

(iv) $\left(\frac{3}{5} \times \frac{-2}{9}\right)^{-2}$ (v) $\dfrac{\left(-\frac{3}{2}\right)^{-1} \times \left(\frac{2}{3}\right)^2}{\left(\frac{2}{3}\right)^{-1} \div \left(\frac{3}{2}\right)}$

6. **Express $(15^3)^{-16}$ as a single exponent of 15.**

7. **By what number should $(7)^{-2}$ be multiplied so that the product may be equal to $(343)^{-1}$?**

8. **By what number should $\left(\frac{-3}{4}\right)^5$ be multiplied so that the product may be equal to $\left(\frac{-64}{27}\right)^{-1}$?**

9. **By what number should $(-512)^{-1}$ be divided so that the quotient may be equal to $(8)^{-2}$?**

10. **By what number should $\left(\frac{144}{225}\right)^{-1}$ be divided so that the quotient may be equal to $\left(\frac{12}{15}\right)^{-4}$?**

11. **Write the following numbers in usual form.**

(i) 5.3×10^5 (ii) 2.9×10^{-10} (iii) 4.6×10^{-12}

(iv) 1.08×10^7 (v) 3.09×10^{11} (vi) 6.00005×10^9.

12. **Write the following numbers in the form $k \times 10^n$ where $1 \le k < 10$ and n is an integer.**

(i) 762850 (ii) 2500000 (iii) 0.09

(iv) 0.0000076 (v) 0.00000008 (vi) 459200000000

(vii) $\dfrac{315 \times 10^5}{0.7 \times 10^3}$ (viii) $\dfrac{4.4 \times 10^{-7}}{44 \times 10^{-5}}$ (ix) $(7 \times 10^2) \times (8 \times 10^3)$

13. **Simplify the following:**

(i) $\left(\frac{7}{8}\right)^{-3} \times \left(\frac{9}{5}\right)^{0} \times 8^{-2} \times \left(\frac{1}{7}\right)^{-1}$

(ii) $\left\{\left[\left(\frac{4}{5}\right)^{3}\right]^{2} \div \left(\frac{4}{5}\right)^{2}\right\} \times \left(\frac{1}{4}\right)^{-2} \times 4^{-1}$

(iii) $\left(-\frac{4}{5}\right)^{3} \times 5^{2} \times \left(-\frac{1}{2}\right)^{5} \times \left(\frac{1}{2}\right)^{-3}$

(iv) $\left[\left(\frac{2}{3}\right)^{2}\right]^{-6} \times \left(\frac{2}{3}\right)^{2} \times \left(\frac{3}{2}\right)^{-10}$

14. **Express the numbers appearing in the following statements in the form $k \times 10^n$ where $1 \le k < n$ and n is an integer.**

(i) The mean distance of the moon from the earth is 384,400,000 metres.

(ii) The distance travelled by a ray of light in one year is 9,460,500,000,000,000 m.

(iii) The number of red blood cells per cubic mm of human blood is approximately 5.5 millions.

15. **Find the value of x so that–**

(i) $\left(\frac{-7}{11}\right)^{-3} \times \left(\frac{-7}{11}\right)^{5x} = \left[\left(\frac{-7}{11}\right)^{-2}\right]^{-1}$

(ii) $\left(\frac{3}{7}\right)^{-2x+1} \div \left(\frac{3}{7}\right)^{-1} = \left[\left(\frac{3}{7}\right)^{-1}\right]^{-7}$

16. **If $\frac{p}{q} = \left(\frac{5}{6}\right)^{-2} \times \left(\frac{4}{3}\right)^{0}$, find the value of $\left(\frac{p}{q}\right)^{-2}$.**

HOTS

1. **Express in exponential form.**

$$\frac{256 \times 81}{64 \times 729}$$

2. **If $2^{2x-3} = (64)^x$, find the value of x.**

ENRICHMENT QUESTION

Find the value of:

(a) $\left(\frac{x}{y}\right)^{a} \times \left(\frac{y}{z}\right)^{a} \times \left(\frac{z}{x}\right)^{a}$

(b) $\left(\frac{x^a}{x^b}\right) \times \left(\frac{x^b}{x^c}\right) \times \left(\frac{x^c}{x^a}\right)$

YOU MUST KNOW

1. If $\frac{p}{q}$ be any rational number and m be any positive integer then,

$$\left(\frac{p}{q}\right)^m = \frac{p^m}{q^m}$$

2. If x be any non-zero rational number and m, n be any positive integers then,

$$x^m \times x^n = x^{m+n} \qquad \text{(Law I)}$$

3. If x be any non-zero rational number and m, n be any positive integers then,

$$x^m \div x^n = x^{m-n} \quad \text{if } m > n \qquad \text{(Law IIa)}$$

4. If x be any non-zero rational number and m, n be any positive integers then,

$$x^m \div x^n = \frac{1}{x^{n-m}} \quad \text{if } m < n \qquad \text{(Law IIb)}$$

5. If x be any rational number other than zero then,

$$x^{\circ} = 1 \qquad \text{(Law III)}$$

6. If x be any non-zero rational number and m, n be any any integer then,

$$(x^m)^n = x^{m \times n} \qquad \text{(Law IV)}$$

7. If x be any rational number other than zero, then x^{-1} denotes the reciprocal of x. (Law V)

8. If x be any rational number other than zero and m be a positive integer. Then,

$$x^{-m} = \frac{1}{x^m}$$

9. If x and y are two rational numbers such that ($x \neq 0$), ($y \neq 0$) and m is any integer then,

$$(x \times y)^m = x^m \times y^m \qquad \text{(Law VI)}$$

10. If x and y are two non-zero rational numbers $y \neq 0$ and m is any integer then,

$$\left(\frac{x}{y}\right)^m = x^m \div y^m \qquad \text{(Law VII)}$$

11. Every number, large or small can be expressed in the form $k \times 10^n$ where k is a terminating decimal satisfying $1 \leq k < 10$ and n an integer, (positive for large numbers and negative for small numbers).

CHAPTER 5

APPLICATION OF PERCENTAGE

INTRODUCTION

Do you remember percentage?

- A fraction with denominator **100** is called **per cent**.
- Symbol used for per cent is **%**

Examples of per cent are–

$$\frac{19}{100} = 19\%, \quad \frac{7}{100} = 7\%$$

Remember

Per cent can be–

(i) converted into a fraction.

(ii) expressed as a ratio.

(iii) converted into a decimal.

Do you know?

The word 'per cent' is an abbreviation of the Latin word **percentum** which means per hundred or hundredths.

Let us do some problems to revive our memory.

1. Express the following per cents in lowest terms.

(i) 4% $\longrightarrow$ $\boxed{4\% = \frac{4}{100} = \frac{1}{25}}$

(ii) $5\frac{1}{4}\%$

(iii) 32%

2. Express the following per cents as decimals.

(i) 33% ⟶ $33\% = \frac{33}{100} = 0.33$

(ii) 1.2%

(iii) 3.25%

3. Write the following fractions as per cents.

(i) $2\frac{3}{4}$ ⟶ $2\frac{3}{4} = \frac{11}{4}$

$\frac{11}{4} = \left(\frac{11}{4} \times 100\right)\% = 275\%$

(ii) $6\frac{1}{2}$

(iii) $\frac{1}{5}$

4. Convert the given ratios into per cents.

(i) 2 : 5 ⟶ $\left(\frac{2}{5} \times 100\right)\% = 40\%$

(ii) 12 : 5

(iii) 13 : 50

5. Write the following decimals as per cents:

(i) 0.34 ⟶ $(0.34 \times 100)\% = 34\%$

(ii) 2.3

(iii) 0.2

6. Find the value of:

(i) 20% of 1 rupee ⟶ 1 rupee = 100 paise

$\frac{20}{100} \times 100 = 20$ paise

(ii) 35% of 500 gm

(iii) 40% of 120 km

SIMPLE APPLICATIONS OF PERCENTAGE

In Class-VI, you have studied how to solve problems on percentage. Here are some more examples to learn the applications of percentage.

Example 1: Radhika spends ₹ 350 every month. If this is 70% of her pocket money, find her pocket money.

Solution: Let Radhika's pocket money = ₹ x

Money spent = ₹ 350

Also money spent is = 70% of x

$\therefore$ 70% of x = 350

$$\Rightarrow \quad \frac{70}{100} \times x = 350$$

$$\Rightarrow \quad x = \frac{350 \times 100}{70}$$

$$\Rightarrow \quad x = 500$$

Hence, Radhika's pocket money is ₹ 500.

Example 2: In a family of 25 persons, 72% read English newspaper, 16% read Hindi newspaper and the rest do not read the newspaper. Find the number of persons who read none of the newspapers.

Solution: Number of persons who read English newspaper = 72% of 25

$$= \frac{72}{100} \times 25$$

$$= 18$$

Number of persons who read Hindi newspaper = 16% of 25

$$= \frac{16}{100} \times 25$$

$$= 4$$

$\therefore$ Number of persons who do not read any of the two newspapers = 25 – (18 + 4)

= 3 persons

Three persons read none of the newspapers.

Example 3: During blood donation camp in a school, 60% of the total students donated blood. If the number of students who did not donate blood was 480, find the total number of students in the school.

Solution: Students who donated blood = 60% of the total students

$\therefore$ Students who did not donate blood = (100 – 60)%

= 40% of the total students

Also, number of students who did not donate blood = 480

$\therefore$ 40% of the total students = 480

$$\Rightarrow \quad \text{Total students} = 480 \times \frac{100}{40}$$

$$= 1200 \text{ students}$$

There are 1200 students in the school.

Worksheet 1

1. A property dealer charges 2% commission from a buyer or a seller of property. If Mr Ram Lal paid ₹ 20,000 as a commission for purchasing a flat, what was the value of the flat?
2. A 3-star hotel in Delhi charges 10% sales tax on the price of the food taken. Mr Saxena and his family had food for ₹ 685. Find the total money he had to pay.
3. A painter paints 240 sq. feet of a wall. If this is 48% of the total wall surface, what is the area of the wall in sq. feet?
4. The salary of a person was increased by 15%. If his salary now is ₹ 9936, what was his initial salary?
5. In a co-educational school, 45% of the total students are girls. If there are 440 boys in the school, find the number of girls in the school.
6. A potato seller sells 70% of the total potatoes and still has 150 kg potatoes left with him. Find the weight of potatoes he had originally.
7. In Rohan's birthday party, 90% of his friends came and three friends did not come. Find the total number of friends Rohan has.
8. In a quiz competition, Ravi gave answers of 70% of the questions. He failed to give answers of six questions. Find the total number of questions asked in the quiz competition.
9. Out of a class of 45 students, five were absent, 30% of the remaining had failed to do homework. Find the number of students who did the homework.

PROFIT AND LOSS

Do you remember S.P. and C.P.?

If S.P. > C.P., then there is a gain or profit.

Profit = S.P. – C.P.

If S.P. < C.P., then there is a loss.

Loss = C.P. – S.P.

Remember

S.P. ⟶ Selling Price

C.P. ⟶ Cost Price

If S.P. = C.P., then there is no profit no loss.

Also, $\text{Profit } \% = \dfrac{\text{Profit}}{\text{C.P.}} \times 100$

$$\text{Loss } \% = \frac{\text{Loss}}{\text{C.P.}} \times 100$$

Remember

The additional amount spend on transportation, rent, labour charges, repairing, etc., are known as **Over Head Expenses.** Over head expenses are included in the cost price.

Let us explain all these with the help of an example.

Example 4: A property dealer purchased a flat for ₹ 5,00,000 and spent ₹ 50,000 on repairing, ₹ 80,000 on wood work and white washing. He, then sold it for ₹ 7,00,000. Find his profit or loss. What was his profit or loss per cent?

Solution:

C.P. of flat = ₹ 5,00,000

Overhead expenses = ₹ 50,000 + ₹ 80,000

Actual C.P. of flat = ₹ (5,00,000 + 50,000 + 80,000) = ₹ 6,30,000

S.P. of flat = ₹ 7,00,000

∴ Profit = ₹ (7,00,000 – 6,30,000)

= ₹ 70,000

$$\text{Profit } \% = \frac{70,000}{6,30,000} \times 100$$

$$= 11\frac{1}{9}\%.$$

Worksheet 2

1. **A shopkeeper purchased a pair of shoes for ₹ 800 and spent ₹ 20 on its box. He sold it at a profit of ₹ 80. Find—**
   (i) Actual C.P. of shoes
   (ii) S.P. of shoes
   (iii) Profit or Loss %
2. **400 mangoes were purchased at ₹ 125 per hundred and sold at a loss of ₹ 100. Find the S.P. of one dozen mangoes.**

3. **A man purchased a cell phone for ₹ 2000. By paying ₹ 200 more, he replaces its body (case). If he sells the cell phone for ₹ 2500, find his profit or loss per cent.**

4. **A man buys 5 dozen eggs at ₹ 36 per dozen. Out of which 5% got broken. He sold the remaining eggs for ₹ 48 per dozen. What was his total gain?**

5. **Rahul buys an almirah for ₹ 2000 and spends ₹ 400 as its transportation charges. If he sells the almirah for ₹ 3000, determine his profit per cent.**

We have learnt how to find profit (or loss) and profit % (or loss %) if C.P. and S.P. of an article is known.

Note:

We can also find S.P. of an article if C.P. is known and vice-versa, provided profit % or loss % is given.

Let us explain this with the help of examples.

Example 5: A shopkeeper purchased a clock for ₹ 300 and sells it at a gain of 20%. Find S.P. of the clock.

Solution:

C.P. of the clock = ₹ 300

Gain % = 20%

Gain = 20% of 300

$$= \frac{20}{100} \times 300 = ₹\ 60$$

S.P. of the clock = C.P. + Gain

= ₹ 300 + ₹ 60

= ₹ 360

Note: We can also calculate S.P. by using formula–

$$\text{S.P.} = \text{C.P.} \times \left[\frac{100 + \text{Gain\%}}{100}\right]$$

$$\text{S.P.} = \text{C.P.} \times \left[\frac{100 - \text{Loss\%}}{100}\right]$$

Example 6: By selling an article for ₹ 475, Rahul lost 5%. Find C.P. of the article.

Solution:

Let C.P. of article = ₹ 100

Loss = 5% of ₹ 100

= ₹ 5

$$\text{S.P. of article} = ₹ (100 - 5)$$
$$= ₹ 95$$
$$\text{If S.P. of article is ₹ 95, then C.P.} = ₹ 100$$
$$\text{If S.P. of article is ₹ 1, then C.P.} = ₹ \frac{100}{95}$$
$$\therefore \quad \text{If S.P. of article is ₹ 475, then C.P.} = ₹ \left(\frac{100}{95} \times 475\right)$$
$$= ₹ 500$$

Note: We can also calculate C.P. using formula–

$$\text{C.P.} = \text{S.P.} \times \left[\frac{100}{100 - \text{Loss}\%}\right]$$

$$\text{C.P.} = \text{S.P.} \times \left[\frac{100}{100 + \text{Gain}\%}\right]$$

Example 7: A man sold two bed sheets at ₹ 600 each. On one he gains 20% and on the other he loses 25%. How much does he gain or lose in the whole transaction?

Solution:

I Case

$$\text{S.P. of bed sheet} = ₹ 600$$
$$\text{Let C.P. of bed sheet} = ₹ 100$$
$$\text{Gain} = 20\% \text{ of } 100$$
$$= ₹ 20$$
$$\therefore \quad \text{S.P.} = ₹ 120$$
$$\text{If S.P. is ₹ 120, then C.P.} = ₹ 100$$
$$\text{If S.P. is ₹ 600, then C.P.} = ₹ \left(\frac{100}{120} \times 600\right)$$
$$= ₹ 500$$

II Case

$$\text{S.P. of bed sheet} = ₹ 600$$
$$\text{Let C.P. of bed sheet} = ₹ 100$$
$$\text{Loss} = \frac{25}{100} \times 100$$
$$= ₹ 25$$
$$\therefore \quad \text{S.P.} = ₹ 75$$
$$\text{If S.P. is ₹ 75, then C.P.} = ₹ 100$$
$$\text{If S.P. is ₹ 600, then C.P.} = ₹ \left(\frac{100}{75} \times 600\right)$$
$$= ₹ 800$$

Note: Now, solve the sum using formula—

$$\text{Total C.P.} = ₹ (500 + 800) = ₹ 1300$$
$$\text{Total S.P.} = ₹ (600 + 600) = ₹ 1200$$
$$\text{Loss} = ₹ 1300 - ₹ 1200 = ₹ 100$$
$$\therefore \quad \text{Loss} = ₹ 100$$

Worksheet 3

1. **I purchased a watch for ₹ 330 and sold it at a loss of 20%. Find S.P. of the watch.**
2. **By selling a T.V. for ₹ 8000, a shopkeeper loses 20% of his cost. If he sells it for ₹ 11000, what profit or loss would be there for him?**
3. **A shopkeeper purchased a T.V. set for ₹ 9000 and sold it at a loss of 5%. Find the selling price of the T.V. set.**
4. **Seeta sells a dining set to Neeta for ₹ 6000 and gains 20%. For how much should she sell it to increase her profit by another 5%?**
5. **Ranjan bought a second hand scooter for ₹ 6000. He spent ₹ 300 on its repairs and sold it to Vineet at a profit of 10%. Vineet sold the scooter to Mukesh at a loss of 10%. At what price did Mukesh buy the scooter?**
6. **A man buys two pens at ₹ 20 each. He sells one at a gain of 5% and other at a loss of 5%. Find his gain or loss per cent.**
7. **Mr. Tandon purchased a computer for ₹ 32000 and a microwave oven for ₹ 6500. On computer he lost 5% and on microwave he gained 15%. Find his total gain or loss per cent.**
8. **Mr. A sells a bicylce to Mr. B at a profit of 20% and Mr. B sells it to Mr. C at a profit of 25%. If Mr. C pays ₹ 1500, what did Mr. A pay for it?**

SIMPLE INTEREST

Do you remember the factors determining simple interest? Let us recall them.

1. **Principal (P)** → The money which we deposit in or borrow from a bank or money lender is called the **principal**.
2. **Rate of Interest (R)** → The interest on ₹ 100 for one year is known as **rate of interest per year** or **rate of interest per annum**.
3. **Time (T)** → The period of time for which the principal is kept in a bank is called **time**.

Simple Interest is the additional money paid by the borrower to the lender for using the money. Simple Interest is also the additional money paid by the bank for the money deposited.

$$\text{Simple Interest} = \text{Principal} \times \text{Rate} \times \text{Time}$$

Now, let us understand how the above mentioned formula is formed.

Example 8: Find the simple interest on ₹ 500 for five years at the rate of 2% per annum?

Solution:

Interest on ₹ 100 for one year = ₹ 2

Interest on ₹ 1 for one year = ₹ $\frac{2}{100}$

Interest on ₹ 500 for one year = ₹ $\frac{2}{100} \times 500$

Interest on ₹ 500 for five years = ₹ $\frac{2 \times 500 \times 5}{100}$

$\therefore$ Simple Interest = Principal × Rate of interest p.a. × Time in years

i.e. S.I. = P × R × T

Simple Interest when added to the principal gives amount.

$\boxed{\text{Amount} = \text{S.I.} + \text{P}}$

Remember

- If time T is in months, then convert it into year by dividing it by 12.
- If time T is in weeks, then convert it into year by dividing it by 52.
- If time T is in days, then convert it into year by dividing it by 365.
- If rate of interest is per rupee and not per cent, then convert it into per cent by multiplying it by 100.

Worksheet 4

1. Find the unknown quantity in each of the following:

Principal	Rate of Interest per annum	Time Period	Simple Interest	Amount
(i) ₹ 400	5%	3 years	_____	_____
(ii) ₹ 450	$4\frac{1}{2}\%$	3 years 4 months	_____	_____
(iii) ₹ 500	$15\frac{1}{2}\%$	146 days	_____	_____
(iv) ₹ 1200	15%	8 April to 20 June	_____	_____

2. **Find the amount from the investment of ₹ 4500 for two years at 5 paise per rupee interest.**

3. **Ramesh took a loan of ₹ 80,000 from a bank at 12% per annum and paid it back after seven months together with interest. Find the amount he paid to the bank.**

4. **Rahul deposited ₹ 7000 at 7% per annum for $4\frac{1}{2}$ years and Rohan deposited ₹ 7000 at 6% per annum for five years. Who will get more interest? What amount will each get?**

5. **Ramit deposited ₹ 80,000 in a bank which pays him 6% interest. After three years, he withdraws the money and buys a car for ₹ 90,000. How much money is left with him?**

We know for getting **Simple Interest**, we require **Principal**, **Rate of Interest** and **Time**.

If out of above four, any three are given to us, we can find the unknown by using the formula (S.I. = P × R × T)

where $R = r\% = \frac{r}{100}$

Remember

$$P = \frac{100 \times S.I.}{r \times T}$$

$$r = \frac{100 \times S.I.}{P \times T}$$

$$T = \frac{100 \times S.I.}{P \times r}$$

We solve some examples to illustrate.

Example 9: Mr Saxena lent a sum of money at $2\frac{1}{2}\%$ per annum simple interest and received an interest of ₹ 2400 in three years. Find the sum.

Solution: Here, Rate of interest $= 2\frac{1}{2}\%$ p.a. $= \frac{5}{2}\%$ p.a.

Simple Interest = ₹ 2400

Time = 3 years

$$\text{Principal} = \frac{100 \times S.I.}{\frac{5}{2} \times 3}$$

$$= \frac{100 \times 2400 \times 2}{5 \times 3}$$

= ₹ 32000

∴ Mr Saxena lent ₹ 32000.

Example 10: A farmer, for purchasing seeds and fertilisers, borrowed a loan from a co-operative bank. After two years he paid ₹ 5434 and settled the account. If the rate of simple interest is $2\frac{1}{4}\%$ per annum, what sum did he borrow?

Solution: Here, Rate of interest $= 2\frac{1}{4}\%$ p.a. $= \frac{9}{4}\%$ p.a.

Time $=$ 2 years

Amount $=$ ₹ 5434

$\therefore$ Principal + Simple Interest $=$ ₹ 5434

$$\Rightarrow \quad P + \left(\frac{P \times r \times T}{100}\right) = 5434$$

$$\Rightarrow \quad P + \left(\frac{P \times 9 \times 2}{4 \times 100}\right) = 5434$$

$$\Rightarrow \quad P + \frac{9}{200}P = 5434$$

$$\Rightarrow \quad \frac{209}{200}P = 5434$$

$$\Rightarrow \quad P = \frac{5434 \times 200}{209}$$

$$P = 5200$$

Therefore the farmer borrowed ₹ 5200 from bank.

Example 11: Simple interest on a sum of money at the end of five years is $\frac{4}{5}$ of the sum itself. Find the rate per cent per annum.

Solution: Let us take sum (Principal) as P

Here, Time $=$ 5 years

Simple Interest $= \frac{4}{5}P$

$$\therefore \quad \frac{P \times r \times T}{100} = \frac{4}{5}P$$

$$\Rightarrow \quad \frac{P \times r \times 5}{100} = \frac{4}{5}P$$

$$\Rightarrow \quad \text{Rate of interest} = \frac{4}{5} \times \frac{100}{5}$$

$$\Rightarrow \quad = 16$$

Therefore, Rate of interest $=$ 16% per annum.

Example 12: At what rate per cent will ₹ 1500 amount to ₹ 2400 in four years?

Solution: Here, Principal $=$ ₹ 1500

Amount $=$ ₹ 2400

$\therefore$ Simple Interest $= 2400 - 1500 = ₹\ 900$

Time $= 4$ years

$$\text{Rate of interest} = \frac{100 \times \text{S.I.}}{\text{P} \times \text{T}} = \frac{100 \times 900}{1500 \times 4} = 15$$

$\therefore$ $= 15\%$

Example 13: In what time will a sum of money double itself at 15% per annum?

Solution: Let Principal $= ₹\ \text{P}$

$\therefore$ Amount $= ₹\ 2\ \text{P}$

$\therefore$ Simple Interest $=$ Amount $-$ Principal $= ₹\ \text{P}$

Rate of interest $= 15\%$ p.a.

$$\text{Time} = \frac{100 \times \text{S.I.}}{\text{P} \times 15}$$

$$= \frac{100 \times \text{P}}{\text{P} \times 15} = \frac{20}{3} \text{ years}$$

$\therefore$ Time $= 6$ years 8 months

Worksheet 5

1. Find the unknown quantity in each of the following:

Principal	Rate of Interest per annum	Time Period	Simple Interest	Amount
(i) ______	12%	$2\frac{1}{2}$ years	₹ 1200	______
(ii) ______	3.5%	2 years	______	₹ 535
(iii) ______	4%	3 years	₹ 120	______
(iv) ₹ 450	______	$3\frac{1}{2}$ years	₹ 189	______
(v) ₹ 850	6%	______	₹ 178.50	______
(iv) ₹ 5400	______	3 years	______	₹ 6210

2. Find the sum of money that amounts to ₹ 5850 in six years at 5% per annum.

3. What sum will earn an interest of ₹ 480 in $2\frac{1}{2}$ years at the rate of 3% per annum simple interest?

4. Mr Mehta borrowed a sum of money at 8% per annum. If he paid ₹ 640 as interest after $5\frac{1}{3}$ years (5 years 4 months), find the sum borrowed by him.
5. Simple interest on a sum of money is $\frac{9}{16}$ of the sum. If the rate is $4\frac{1}{2}\%$ per annum, find the time.
6. At what rate per cent per annum will a sum of money double itself in eight years?
7. Rahul borrowed ₹ 50,000 from a Bank on 1 March 2014 and paid ₹ 53150 on 6 October 2014. Find the rate of interest charged by the Bank.
8. In what time will a sum of money double itself at 15% per annum?
9. In what time will the Simple Interest on ₹ 400 at 10% per annum be the same as the Simple Interest on ₹ 1000 for four years at 4% per annum?
10. Mr Jane donates ₹ 1 lakh to a school and the interest on it is to be used for awarding five scholarships of equal value. If the value of each scholarship is ₹ 1,500, find the rate of interest.

VALUE BASED QUESTIONS

1. In a survey it was found that out of 125 people in a park, 12% jog, 16% do yoga and rest prefer to walk.
   (a) Find the number of people who prefer to walk.
   (b) Discuss the importance of exercises like jogging, yoga, walking, etc.
2. The library teacher of a school keeps record of students of each class about their reading habits. The results of Class-VII students for the month of April are as follows–

Books read	Percentage
0	14
1 – 3	28
4 – 6	26
More than 6	32

   (a) If total number of students of Class-VII is 500, how many students had read books in the month of April?
   (b) What is the importance of reading books?

BRAIN TEASERS

1. **A. Tick (✓) the correct option.**

(a) 12% of 50 + 5% of 120 is equal to–

(i) 10 (ii) 15 (iii) 12 (iv) 20

(b) What is 50% of a number whose 200% is 20?

(i) 15 (ii) 5 (iii) 20 (iv) 10

(c) At what rate of Interest per annum should Rita invest if she wants to grow her ₹ 2500 to ₹ 4000 in five years?

(i) 10% (ii) 15% (iii) 12% (iv) 13%

(d) Rohan gave the pizza delivery person a tip of ₹ 30, which was 20% of his total bill for the pizza ordered. How much did pizza cost him?

(i) ₹ 150 (ii) ₹ 180 (iii) ₹ 140 (iv) ₹ 200

(e) In a country, there are 215 highway accidents associated with drinking alcohol. Out of these, 113 are caused by excessive speed. Approximately what per cent of accidents are speed related?

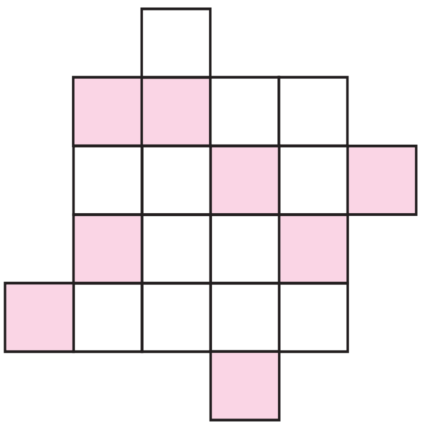
(i) 47% (ii) 49% (iii) 51% (iv) 53%

B. Answer the following questions.

(a) What per cent of numbers from 1 to 20 are divisible by 4?

(b) A child had certain amount of chocolates. He ate 35% of the chocolates but still had 13 chocolates. What is the number of chocolates the child originally had?

(c) How many more squares must be shaded so that only 30% of the figure is left unshaded?



(d) A bucket of 20 litres is 35% full. How many litres of water must be put into it so that it is 65% full?

(e) Mr. Sanjay deposited ₹ 15000 in the bank for his five-year old daughter as he wishes to give his daughter the amount of ₹ 21000 on her thirteenth birthday. At what rate of interest should the money be invested?

2. Rahul sold a watch to Sohan at a gain of 10% and Sohan sold it to Mohan at a loss of 10%. If Mohan paid ₹ 990 for it, find the price paid by Rahul.

3. In a school, there are 50 teachers; 30% of them are men and the rest are women. If 60% of the male teachers are married, find the number of married male teachers.

4. Solve.

(i) If x% of y is 13x, then find the value of y.

(ii) Find $12\frac{1}{2}$ % of $3\frac{1}{2}$ % of ₹ 256.

5. In an exam, 14% students failed and 559 students passed. Determine the number of students who failed.

6. A student needs to score at least 30% marks to pass a test. If a student gets 135 marks and fails by 15 marks, find the maximum marks of the test.

7. A man bought cardboard sheet for ₹ 3600 and spent ₹ 100 on transport. Paying ₹ 300 for labour, he had 330 boxes made, which he sold at ₹ 14 each. Find the profit per cent.

8. The S.P. of an article is three-fourth of its C.P. What is the loss per cent?

9. In what time will the Simple Interest on a certain sum be three-fourth of the principal at 6% per annum?

10. At what rate of interest per annum will a sum of ₹ 8000 amount to ₹ 9260 in $3\frac{1}{2}$ years?

11. If 40% of 70 is x more than 30% of 80, then find x.

12. A manufacturer sells three products A, B, C.

Product A costs ₹ 200 and is sold for ₹ 260.

Product B costs ₹ 150 and is sold for ₹ 180.

Product C costs ₹ 100 and is sold for ₹ 140.

Which product should be manufactured more in order to have maximum profit percentage?

13. Find x:

48% of 480 + 25% of 250 – x = 200.

HOTS

There are 40 papers of students of a class to be checked by two teachers. Mr. Ashok can check five papers in an hour and Mrs. Meena can check four papers in an hour. If Mr. Ashok spends three hours in checking the papers and Mrs. Meena works for two hours, what percentage of the papers will be checked in all?

YOU MUST KNOW

1. If S.P. > C.P., then there is profit.

   Profit = S.P. – C.P.

2. If C.P. > S.P., then there is loss.

   Loss = C.P. – S.P.

3. Over head Expenses are included in the C.P.

4. Profit per cent or Loss per cent are always calculated on C.P. (i.e. price, over head expenses included).

5. Profit % = $\frac{\text{Profit}}{\text{C. P.}} \times 100$

6. Loss % = $\frac{\text{Loss}}{\text{C. P.}} \times 100$

7. (a) S.P. = $\text{C. P.} \times \left[\frac{100 + \text{Gain }\%}{100}\right]$

   (b) S.P. = $\text{C. P.} \times \left[\frac{100 - \text{Loss }\%}{100}\right]$

8. (a) C.P. = $\text{S. P.} \times \left[\frac{100}{100 + \text{Gain }\%}\right]$

   (b) C.P. = $\text{S. P.} \times \left[\frac{100}{100 - \text{Loss }\%}\right]$

9. Simple Interest (S.I.) = P × R × T

where P = Principal

R = Rate of interest per annum = r%

T = Time (in years).

10. Amount = Principal + Interest

11. $P = \dfrac{S.I. \times 100}{r \times T}$

$R = \dfrac{S.I. \times 100}{P \times T}$

$T = \dfrac{S.I. \times 100}{P \times r}$

CHAPTER 6

ALGEBRAIC EXPRESSIONS

INTRODUCTION

You have studied algebraic expressions in Class-VI. Do you remember what an algebraic expression is? An **algebraic expression** is a combination of constants and variables connected by means of four fundamental operations (+, –, × and ÷). For instance, $2x + 3$, $8a^2b + a^3b^3 - 5a$, $10y^4$, $\frac{1}{4}x - \frac{3}{2}z$ are algebraic expressions. You may also recall that:

1. Parts of an algebraic expression separated by the symbols + or – are called **terms of an algebraic expression**.
2. An algebraic expression having only one term is called a **monomial**.
3. An algebraic expression having two terms is called a **binomial**.
4. An algebraic expression having three terms is called a **trinomial**.

You know addition and subtraction of algebraic expressions.

Let us look at the following examples to brush up our memory.

Example 1: Add $-8x^2 + 6y^2 + 5$ and $2x^2 - 3y^2$.

Solution: Adding by **column method:**

$$\begin{array}{r} -8x^2 + 6y^2 + 5 \\ +\ 2x^2 - 3y^2 \quad\;\; \\ \hline -6x^2 + 3y^2 + 5 \\ \hline \end{array}$$

(Observe that the like terms are written below each other and then they are added)

Now, adding by **horizontal method:**

$(-8x^2 + 6y^2 + 5) + (2x^2 - 3y^2)$

$= (-8x^2 + 2x^2) + (6y^2 - 3y^2) + 5$ (Like terms are combined together)

$= -6x^2 + 3y^2 + 5$

Example 2: Subtract $(6p^2 - 13pq + r)$ from $(-12p^2 + 5pq - 14r)$

Solution: Subtracting by **column method:**

$$\begin{array}{rrr} -12p^2 & +\ 5pq & -\ 14r \\ 6p^2 & -\ 13pq & +\ r \\ - & + & - \\ \hline -18p^2 & +\ 18pq & -\ 15r \\ \hline \end{array}$$

{ We change the signs of the subtrahend and add it to minuend }

Subtracting by **horizontal method:**

$(-12p^2 + 5pq - 14r) - (6p^2 - 13pq + r)$

$= -12p^2 + 5pq - 14r - 6p^2 + 13pq - r$

$= (-12p^2 - 6p^2) + (5pq + 13pq) + (-14r - r)$ { Changing the signs of the terms }

$= -18p^2 + 18pq - 15r$

Now, we will learn how to multiply the algebraic expressions.

MULTIPLICATION OF MONOMIALS

First of all, we will study multiplication of monomials. Let us understand it with the help of an example.

Example 1: Multiply $5x^2y$ by $4xy$.

Solution: Constants involved here are 5 and 4.

Variables involved here are x^2 and y (first monomial) and x and y (second monomial).

Product of $5x^2y$ and $4xy$ is written as

$= (5 \times 4) \times (x^2 \times x) \times (y \times y)$

$= 20x^3y^2$

So, the steps for finding the product of monomials are:

Step I: Form the groups of constants and variables occurring in the monomials.

Step II: Multiply the constants involved.

Step III: Multiply the like variables.

Step IV: Write the product of constants and variables.

Let us illustrate the above steps with the help of an example.

Example 2: Solve $(-3ab) \times (7a^2c)$.

Solution:

Step I: $(-3) \times 7 = -21$

Step II: $a \times a^2 = a^3$

Step III: $(-21) \times a^3 \times b \times c = -21a^3bc$

Thus, $(-3ab) \times (7a^2c) = -21a^3bc$

$$\boxed{-3}\ a\ b \times \boxed{7}\ a^2\ c$$

$$= -21 \times a^3 \times b \times c$$

$$= -21\,a^3\,bc$$

Note:

For finding the product of more than two monomials, you will follow the same steps.

Example 3: Find the product $\left(6x^2y\right)\times\left(\frac{2}{3}xy^2\right)\times\left(-5yz^2\right)$.

Solution:

Step I: $6 \times \frac{2}{3} \times -5 = -20$

Step II: $x^2 \times x = x^3$

Step III: $y \times y^2 \times y = y^4$

Step IV: $(-20) \times x^3 \times y^4 \times z^2 = -20x^3y^4z^2$

So, $\left(6x^2y\right)\times\left(\frac{2}{3}xy^2\right)\times\left(-5yz^2\right) = -20x^3y^4z^2$

Note:

Since the order in which numbers are added or multiplied does not affect the sum or product of numbers, monomials can be arranged in any order while multiplying.

Example 4: Find the product $\left(5x^2y\right)\times\left(-\frac{3}{5}y^2z\right)\times\left(2xz^2\right)$. Also verify the result for $x = 1$, $y = -1$ and $z = 2$.

Solution:

$$\left(5x^2y\right)\times\left(-\frac{3}{5}y^2z\right)\times\left(2xz^2\right) = \left(5\times-\frac{3}{5}\times 2\right)\times\left(x^2\times x\right)\times\left(y\times y^2\right)\times\left(z\times z^2\right)$$

$$= -6x^3y^3z^3$$

Therefore, $\left(5x^2y\right)\times\left(-\frac{3}{5}y^2z\right)\times\left(2xz^2\right) = -6x^3y^3z^3$ .... (I)

When $x = 1$, $y = -1$, $z = 2$, let us verify the equation (I).

$$\text{L.H.S.} = \left(5\times 1^2\times(-1)\right)\times\left(-\frac{3}{5}\times(-1)^2\times 2\right)\left(2\times 1\times 2^2\right)$$

$$= -5\times\left(-\frac{6}{5}\right)\times 8 = 48$$

$$\text{R.H.S.} = -6\,(1)^3\,(-1)^3\,(2)^3$$

$$= -6\,(1)\,(-1)\,(8) = 48$$

$$\therefore\quad \text{L.H.S.} = \text{R.H.S.}$$

APPLICATION OF PRODUCT OF MONOMIALS

Consider a rectangle with length l and breadth b. You can make out that l and b are monomials. What about the area of the rectangle? It is the product of the two monomials l and b, and is written as lb. Now consider a rectangle whose breadth is x and length is twice the breadth, i.e. $2x$. You can see that area of the rectangle is the product of monomials x and $2x$, ie. $2x^2$.

Example 5: Find the area of the rectangle whose length is y and breadth is half of its length.

Solution: Area of rectangle = Length × Breadth

$$= y\times\frac{y}{2}$$

$$= \left(1\times\frac{1}{2}\right)\times(y\times y)$$

$$= \frac{1}{2}y^2 \text{ (square units)}$$

Worksheet 1

1. **Find the product:**

   (i) $9x^3 \times 2x^4$ (ii) $-6a^2 \times 5a^7$ (iii) $-8y^9 \times -4y^3$

2. **Multiply the monomials:**

   (i) $7\,pq$ and $\frac{4}{3}\,p^2q^3$ (ii) $12a^2b^6c^8$ and $-3a^7b^4c^3$ (iii) $\frac{2}{5}\,x^2y$ and $\frac{5}{3}\,x^3y^2z^2$

3. **Multiply the monomials:**

   (i) $3x^7$, $4x^2$ and $-5x^3$ (ii) $1.2\,a^2b^2$, $5ab^4c^2$ and $1.1a^5bc^7$

   (iii) $\frac{3}{4}\,pq$, $\frac{1}{2}\,qr^2$, $-5p^2r^3$ and $-6r^5$

4. Find the product of $\left(\frac{1}{2}x^3\right)(-10x)\left(\frac{1}{5}x^2\right)$ and verify the result for $x = 1$.

5. Find the product of $(-3xyz)\left(\frac{4}{9}x^2z\right)\left(-\frac{27}{2}xy^2z\right)$ and verify the result for $x = 2$, $y = 3$ and $z = -1$.

6. Find the product of $(a^2bc^2)\ (9ab^2c^2)\ (-4ab^2c^4)$ and verify the result for $a = \frac{1}{2}$, $b = -1$, $c = 1$.

7. Find the area of a rectangle whose sides are $2a$ and $3a$.

8. Find the area of a rectangle whose length is thrice its breadth where, breadth is $4x$.

9. Find the area of a rectangle whose breadth is b and length is square of breadth.

MULTIPLICATION OF A MONOMIAL AND A BINOMIAL

Recall that a **binomial** is an algebraic expression having two terms. You may look upon a binomial as sum or difference of two monomials having unlike terms, e.g. binomial $3xy + 2y^2$ is the sum of monomials $3xy$ and $2y^2$.

Now, to illustrate multiplication of a monomial and a binomial, we consider the following examples.

Example 6: Multiply $3xy$ and $8x^2 + 7xy$.

Solution: $3xy(8x^2 + 7xy) = 3xy \times 8x^2 + 3xy \times 7xy$

$= 24x^3y + 21x^2y^2$

Thus, to multiply a monomial and a binomial we take the following steps:

Step I: Multiply the monomial with first term of the binomial.

Step II: Multiply the monomial with second term of the binomial.

Step III: Add the products obtained in Step I and Step II if the terms of the binomial are separated by '+' sign. Subtract the terms if they are separated by '–' sign.

Example 7: Find the product $2a^2(9b^2 + 5ab)$.

Solution: **Step I:** $2a^2 \times 9b^2 = 18a^2b^2$

Step II: $2a^2 \times 5ab = 10a^3b$

Step III: $18a^2b^2 + 10a^3b$

So, $2a^2(9b^2 + 5ab) = 18a^2b^2 + 10a^3b$

Note:

A monomial 'A' is multiplied by a binomial 'B + C' as A (B + C) = AB + AC, just like numbers are multiplied by using distributive property.

Example 8: Find the product $4y^2\,(xy - x^2)$ and then evaluate it for $x = -2$, $y = 3$.

Solution: **Step I:** $4y^2 \times xy = 4xy^3$

Step II: $4y^2 \times x^2 = 4x^2y^2$

$4y^2\,(xy - x^2)$

Step III: $4xy^3 - 4x^2y^2$

So, $4y^2\,(xy - x^2) = 4xy^3 - 4x^2y^2$ ... (I)

When $x = -2$, $y = 3$,

$$4xy^3 - 4x^2y^2 = 4 \times (-2) \times 3^3 - 4 \times (-2)^2 \times 3^2$$

$$= -216 - 144 = -360$$

Example 9: Find the product $\frac{1}{4}a^2b\left(5b^2 - 3ab\right)$ and verify the result for $a = 2$ and $b = -1$.

Solution: **Step I:** $\frac{1}{4}a^2b \times 5b^2 = \frac{5}{4}a^2b^3$

Step II: $\frac{1}{4}a^2b \times 3ab = \frac{3}{4}a^3b^2$

$\boxed{\frac{1}{4}a^2b\,(5b^2 - 3ab)}$

Step III: $\frac{5}{4}a^2b^3 - \frac{3}{4}a^3b^2$

So, $\frac{1}{4}a^2b\left(5b^2 - 3ab\right) = \frac{5}{4}a^2b^3 - \frac{3}{4}a^3b^2$ ... (I)

When $a = 2$ and $b = -1$, we evaluate both sides of equation (I)

$$\text{L.H.S.} = \frac{1}{4} \times 2^2 \times (-1)\,[5 \times (-1)^2 - 3 \times 2 \times (-1)]$$

$$= \frac{1}{4} \times 4 \times (-1)\,(5 \times 1 + 6)$$

$$= -1\,(5 + 6) = -11$$

$$\text{R.H.S.} = \frac{5}{4} \times 2^2 \times (-1)^3 - \frac{3}{4} \times 2^3 \times (-1)^2$$

$$= \frac{5}{4} \times 4 \times (-1) - \frac{3}{4} \times 8 \times 1 = -5 - 6 = -11$$

$\therefore$ L.H.S. = R.H.S.

Example 10: Simplify $2a\left(3a^2-6b\right)-\frac{1}{2}b\,(2ab+8a)$

Solution: Let us first solve the product $2a\left(3a^2-6b\right)$.

[This example has been taken to show how two products are combined having negative sign between them.]

$$2a\left(3a^2-6b\right) = 2a \times 3a^2 - 2a \times 6b$$

$$= 6a^3 - 12ab$$

Now, solving the product $\frac{1}{2}b\,(2ab+8a)$, we have

$$\frac{1}{2}b\,(2ab+8a) = \frac{1}{2}b \times 2ab + \frac{1}{2}b \times 8a$$

$$= ab^2 + 4ab$$

Now, putting both the products together,

$$2a\left(3a^2-6b\right)-\frac{1}{2}b\,(2ab+8a)$$

$$= \left(6a^3-12ab\right)-\left(ab^2+4ab\right)$$

$$= 6a^3 - 12ab - ab^2 - 4ab$$

$$= 6a^3 - ab^2 + (-12ab - 4ab)$$ [Putting the like terms together]

$$= 6a^3 - ab^2 + (-16ab)$$

$$\therefore \quad 2a\left(3a^2-6b\right)-\frac{1}{2}b\,(2ab+8a)$$

$$= 6a^3 - ab^2 - 16ab$$

Example 11: Simplify $p^2\,(2pq + q^3) - 2q^2\,(p^2q + 5)$

Solution: $p^2\,(2pq + q^3) - 2q^2\,(p^2q + 5)$

$$= p^2 \times 2pq + p^2 \times q^3 - 2q^2 \times p^2q - 2q^2 \times 5$$

$$= 2p^3q + \underbrace{p^2q^3 - 2p^2q^3} - 10q^2$$

$$= 2p^3q - p^2q^3 - 10q^2$$

Worksheet 2

1. **Find the product.**

   (i) $(x^2 + 3xy)(4x)$ (ii) $9\,pqr\,(2p^2 q - 3q^2 r)$ (iii) $\frac{3}{4}a^2\left(\frac{2}{3}b^2 + 8ab\right)$

2. **Find the following products and then evaluate when $x = 2$, $y = -1$**

   (i) $(7x + 9y^2)(3xy^2)$ (ii) $-11x\left(2y^5 - \frac{3}{11}x^2 y^3\right)$ (iii) $0.5x\,(2x^2y^2 + 1.5xy^3)$

3. **Find the product of $8s^2(t^2 - 2st)$ and verify the result when $s = 1$ and $t = 5$.**

4. **Find the product of $\frac{2}{7}x^2$ $(7y + 14x)$ and verify the result when $x = 2$ and $y = 3$.**

5. **Find the product of $0.2xy\,(3x + 2y)$ and verify the result when $x = 5$ and $y = -1$.**

6. **Simplify:**

   (i) $ab(a^2 - b^2) + b^3(a - 2b)$

   (ii) $-6x^2(xy + 2y^2) - 3y^2(2x^2 + y)$

   (iii) $2x(x - y^2) - 3y(xy + 2x) - xy(x + y)$

MULTIPLICATION OF BINOMIALS

You are now familiar with mutliplication of a monomial and a binomial. Do you remember how to evaluate $(a + b)(c + d)$ where, a, b, c, d are numbers? Recall that you do it by using distributive property twice as shown in the following:

$$(a + b)(c + d) = a(c + d) + b(c + d)$$

$$= ac + ad + bc + bd \quad \text{... (1)}$$

or, $$(a + b)(c + d) = (a + b)c + (a + b)d$$

$$= ac + bc + ad + bd \text{ which is same as (1)}$$

You will use the same property for multiplication of binomials. Let us take an example.

Example 12: Find the product of $(x + 3y)$ and $(2x + y)$.

Solution: $(x + 3y)(2x + y) = x(2x + y) + 3y(2x + y)$

$$= 2x^2 + xy + 6xy + 3y^2$$

$$= 2x^2 + 7xy + 3y^2$$

The product clearly shows that each term of the first binomial is multiplied with each term of the second binomial and the products are then added.

So, to multiply two binomials $A + B$ and $C + D$, the steps to be followed are:

Step I: Multiply A and $(C + D)$ to get $AC + AD$.

Step II: Multiply B and $(C + D)$ to get $BC + BD$.

Step III: Add the products obtained from Step I and Step II, as

$AC + AD + BC + BD$.

Step IV: Combine like terms (if any).

Example 13: Find the product of $(2pq - q^2)$ and $(3p^2 + 4q)$ and verify the result when $p = 2$ and $q = -2$.

Solution: Here, $A = 2pq$, $B = -q^2$, $C = 3p^2$ and $D = 4q$

Step I: $2pq\,(3p^2 + 4q) = 6p^3q + 8pq^2$

Step II: $-q^2\,(3p^2 + 4q) = -3p^2q^2 - 4q^3$

Step III: $(6p^3q + 8pq^2) + (-3p^2q^2 - 4q^3) = 6p^3q - 3p^2q^2 + 8pq^2 - 4q^2$

So, $(2pq - q^2)\,(3p^2 + 4q) = 6p^3q - 3p^2q^2 + 8pq^2 - 4q^2$

Now, to verify the result, put $p = 2$, $q = -2$ in L.H.S and R.H.S.

$$\text{L.H.S.} = [2 \times 2 \times (-2) - (-2)^2]\,[3 \times 2^2 + 4 \times (-2)]$$

$$= (-8 - 4)\,(12 - 8) = -12 \times 4 = -48$$

$$\text{R.H.S.} = 6 \times 2^3 \times (-2) + 8 \times 2 \times (-2)^2 - 3 \times 2^2 \times (-2)^2 - 4 \times (-2)^3$$

$$= -96 + 64 - 48 + 32 = -48$$

$\therefore$ L.H.S. = R.H.S.

Example 14: Simplify $(2a^2 + 5b^2)\,(a - b) + (a^2 - b^2)\,(3a + 4b)$

Solution: $(2a^2 + 5b^2)\,(a - b) + (a^2 - b^2)\,(3a + 4b)$

$$= (2a^2 + 5b^2)a - (2a^2 + 5b^2)b + (a^2 - b^2)3a + (a^2 - b^2)4b$$

$$= 2a^3 + 5ab^2 - 2a^2b - 5b^3 + 3a^3 - 3ab^2 + 4a^2b - 4b^3$$

$$= 5a^3 + 2a^2b + 2ab^2 - 9b^3$$

Worksheet 3

1. Find the product of the following binomials.

(i) $(5x + 3)(2x + 4)$ (ii) $(7p - 3q)(2p + 5q)$ (iii) $\left(\frac{2}{5}a + b\right)\left(a^2 - \frac{1}{5}b^2\right)$

(iv) $(m^2 n - 5)(6 - mn^2)$ (v) $(4x^2 + 7y^3)(3xy - 2y^2)$ (vi) $(1.1x + 2.7y)(1.1x - 2.7y)$

2. Multiply the following binomials and verify the results for the given values.

(i) $(2x^2 - 5y)(5x + 2y^2)$; $x = 2,\ y = -1$

(ii) $\left(-\frac{1}{4}a + \frac{1}{5}b\right)\left(\frac{1}{4}a + \frac{1}{5}b\right)$; $a = 8,\ b = 5$

(iii) $(m + n)(2m - 3n)$; $m = -2,\ n = 0$

(iv) $(0.1p + 0.2q)(0.2q - 0.1p)$; $p = 10,\ q = 5$

3. Simplify the following:

(i) $(2x - 3y)(3x + y) + (x + 2y)(x - y)$

(ii) $\left(\frac{2}{3}x + 4\right)\left(\frac{3}{2}x + 6\right) - \left(\frac{1}{7}x - 1\right)\left(\frac{1}{7}x + 1\right)$

(iii) $(7abc + b^2)(7 - bc) + c(2b^3 - 9ab)$

(iv) $(p + q)(p^2 - q^2) + (p - q)(p^2 + q^2)$

(v) $(x + 2y)(-3x - y) - (x + y)(x - y) + (x - 2y)(-2x + y)$

(vi) $(a^2 + b^2)(a^2 + b^2) - (a^2 - b^2)(a^2 - b^2)$

MULTIPLICATION OF A BINOMIAL AND A TRINOMIAL

Do you remember what a trinomial is? It is an algebraic expression containing three terms. For instance, $9x^2 - 5y^2 + y$. You can look upon a trinomial as sum of three monomials with unlike terms. ($9x^2$, $-5y^2$ and y in the above example)

To multiply a binomial of the type $(A + B)$ and a trinomial of the type $(C + D + E)$, we follow the following steps.

Step I: Multiply A by the trinomial $(C + D + E)$.

Step II: We get sum of the products of the form $AC + AD + AE$.

Step III: Now, multiply B by the trinomial $(C + D + E)$.

Step IV: We get sum of the products as $BC + BD + BE$.

Step V: Adding the products obtained in Steps II and IV as $AC + AD + AE + BC + BD + BE$.

Step VI: Combine all the like terms, if they exist.

Example 15: Find the product $(3x + 1)(x^3 + 2x^2 - 1)$.

Solution: Here, $A = 3x$, $B = +1$, $C = x^3$, $D = 2x^2$ and $E = (-1)$.

So, sum of the products as in Step II above will be

Step I: $AC + AD + AE$

$= 3x \times x^3 + 3x \times 2x^2 + 3x \times (-1)$

$= 3x^4 + 6x^3 - 3x$

Now, sum of the products as in Step IV above will be

Step II: $BC + BD + BE$

$= 1 \times x^3 + 1 \times 2x^2 + 1 \times (-1)$

$= x^3 + 2x^2 - 1$

Now, adding both the sum of the products,

Step III: $AC + AD + AE + BC + BD + BE$

$= 3x^4 + 6x^3 - 3x + x^3 + 2x^2 - 1$

$= 3x^4 + \underbrace{(6x^3 + x^3)}_{\text{(Combining the like terms)}} + 2x^2 - 3x - 1$

$= 3x^4 + 7x^3 + 2x^2 - 3x - 1$

To multiply a binomial and a trinomial we again make use of distributive property. Let us understand it with the help of an example.

Example 16: Multiply $x - 4y$ and $5x^2 - 2xy + y^2$.

Solution: **Step I:** $x(5x^2 - 2xy + y^2) = 5x^3 - 2x^2y + xy^2$.

Step II: $-4y(5x^2 - 2xy + y^2) = -20x^2y + 8xy^2 - 4y^3$

Step III: $(5x^3 - 2x^2y + xy^2) + (-20x^2y + 8xy^2 - 4y^3)$

Step IV: Combining like terms we get $5x^3 - 22x^2 + 9xy^2 - 4y^3$

So, $(x - 4y)(5x^2 - 2xy + y^2) = 5x^3 - 22x^2y + 9xy^2 - 4y^3$

Example 17: Find the product $(a - b)(a^2 + ab + b^2)$ and verify the result for $a = 2$, $b = -3$.

Solution: **Step I:** $a(a^2 + ab + b^2) = a^3 + a^2b + ab^2$

Step II: $-b(a^2 + ab + b^2) = -a^2b - ab^2 - b^3$

Step III: $(a^3 + a^2b + ab^2) + (-a^2b - ab^2 - b^3)$

Step IV: $a^3 - b^3$

So, $(a - b)(a^2 + ab + b^2) = a^3 - b^3$

When $a = 2$, $b = -3$, then

$$\begin{aligned} \text{L.H.S.} &= [2 - (-3)][2^2 + 2 \times (-3) + (-3)^2] \\ &= 5(4 - 6 + 9) = 5 \times 7 = 35 \\ \text{R.H.S.} &= 2^3 - (-3)^3 \\ &= 8 - (-27) = 8 + 27 = 35 \\ \therefore \quad \text{L.H.S.} &= \text{R.H.S.} \end{aligned}$$

Worksheet 4

1. Find the following products.

(i) $(x + 3)(x^2 + 2x - 1)$

(ii) $(7y - 2)(5y^2 - 3y + 2)$

(iii) $(p^3 + 3p + q)(9p + 2q)$

(iv) $(-2x^2 + xy - y^2)(3x + 4y)$

(v) $\left(\frac{2}{5}a + \frac{1}{7}b\right)(3a + 4b - 2)$

(vi) $(0.1a - 0.2c)(a + c + ac)$

2. Simplify the following and verify the results for the given values.

(i) $(x^2 - 4xy + y^2)(x - 2y)$; $x = 3$, $y = 2$

(ii) $(7x^2y - 3z^2)(x + y + z)$; $x = 1$, $y = 1$, $z = -1$

(iii) $(4a^2 - 6ab + 9b^2)(2a + 3b)$; $a = 2$, $b = 1$

(iv) $(m^2 - 10m + 25)(m - 5)$; $m = -2$

(v) $(p^2 + q^2 + r^2)(pq + qr)$; $p = 2$, $q = -3$, $r = 1$

(vi) $\left(\frac{5}{4}x^2 - \frac{3}{2}xy\right)\left(x + y + y^2\right)$; $x = 2$, $y = 2$

FACTORISATION OF ALGEBRAIC EXPRESSIONS

You already have a concept of factors and multiples. Let us recall that we can find the prime factors of 30 as 2, 3, 5 and 30 can be expressed as the product of its prime factors. Thus, $30 = 2 \times 3 \times 5$.

So, we can express any number as a product of two or more prime factors. Similarly, to find the factors of an algebraic expression, we shall write it as the product of two or more algebraic expressions. Each expression is known as the **factor** of the given algebraic expression. So, we can conclude that the process of writing a given algebraic expression as a product of two or more algebraic factors is known as **factorisation**.

FACTORS OF ALGEBRAIC EXPRESSIONS

To understand the concept of factorisation of algebraic expression, we begin with the simplest algebraic expression which is a monomial and write it as the product of two more factors.

Let us illustrate it with the help of an example.

Example 18: Write $8a^2b$ in the form of product of some factors.

Solution:

$8a^2b = 1 \times 8a^2b$	$8a^2b = 2a^2 \times 4b$
$8a^2b = 8 \times a^2b$	$8a^2b = 2b \times 4a^2$
$8a^2b = 8a \times ab$	$8a^2b = 4ab \times 2a$
$8a^2b = 8ab \times a$	$8a^2b = 2ab \times 4a$
$8a^2b = 8a^2 \times b$	$8a^2b = 2 \times 2ab \times 2a$
$8a^2b = a^2 \times 8b$	

So, 1, $8a^2b$; 8, a^2b; $8a$, ab; $8ab$, a; $8a^2$, b; a^2, $8b$; $2a^2$, $4b$; $2b$, $4a^2$; $4ab$, $2a$; $2ab$, $4a$; 2 are some possible factors of $8a^2b$.

COMMON FACTORS OF MONOMIALS

To understand the term **common factors,** let us write some factors of $4xy$ and $9x$.

Factors of $4xy$ are 1, $4xy$, 4, x, $4x$, y, $4y$, xy, $2x$, $2y$, $2xy$, 2.

Factors of $9x$ are 1, x, 9, $9x$, 3, $3x$.

Factors common to both $9x$ and $4xy$ are 1 and x. We, therefore, infer that the factors which are common to both monomials are **common factors**. It means that these factors will occur in both the monomials.

Example 19: Find the common factors of $3x^2$ and $6xy$.

Solution: Factors of $3x^2$ are 1, $3x^2$, 3, x^2, $3x$, x.

Factors of $6xy$ are 1, $6xy$, 6, xy, $6x$, y, $6y$, x, 2, $3xy$, 3, $2xy$, $3x$, $2y$, $2x$, $3y$.

The common factors of both monomials are 1, x, 3, $3x$.

HIGHEST COMMON FACTOR OF MONOMIALS

Highest Common Factor (H.C.F.) of given monomials is a common factor having greatest coefficient and highest power of the variable. To understand the concept more clearly, let us take an example.

Example 20: Find the H.C.F. of $4a^3b^3$ and $12ab^2c$.

Solution: Common factors of $4a^3b^3$ and $12ab^2c$ are 1, 2, 4, a, b, $4a$, $4b$, ab, $4ab$, b^2, $4b^2$, ab^2, $4ab^2$. Of all these, let us consider the factor $4ab^2$.

1. Here, 4 represents the H.C.F. of numerical coefficients of two given monomials 4 and 12.
2. a is the highest common power of variable a and b^2 is the highest common power of the variable b, in the two monomials.

So $4ab^2$ is the H.C.F. of $4a^3b^3$ and $12ab^2c$.

Remember

To find the H.C.F. of two or more monomials, we follow the given steps:

- find the numerical coefficients and calculate their H.C.F.
- find the common variables appearing in the given monomials.
- find the highest common power of each variable in the given monomials.
- the product of H.C.F. of numerical coefficients and the highest common powers of the variables gives the H.C.F. of the monomials.

Example 21: Find the H.C.F. of $21x^3y^7$ and $35x^5y^5$.

Solution: H.C.F. of 21 and 35 is 7.

Common variables here are x and y.

Highest common power of x in two monomials = x^3

Highest common power of y in two monomials = y^5

So H.C.F. = $7x^3y^5$

FACTORISATION BY TAKING OUT A COMMON FACTOR

We know that an algebraic expression is a sum or difference of monomials. Let us take up an example to find the factorisation of algebraic expression consisting of a common monomial.

Example 22: Factorise the binomial $3x^2y - 6xy^2$.

Solution: We know H.C.F. shall be the product of the highest common coefficient and highest common powers of the variables respectively.

H.C.F. of $3x^2y$ and $6xy^2$ is (3) (xy) or $3xy$.

So, $3x^2y - 6xy^2 = 3xy\,(x - 2y)$.

$3xy$ and $(x - 2y)$ are the two factors of the given expression.

$3x^2y - 6xy^2$

The coefficient of two monomials is 3 and 6 respectively and their H.C.F is 3.

The highest common power of x and y is x and y.

Remember

For factorisation by taking out common factor–

- find the H.C.F. of all the terms in an expression.
- express each term of the given expression as a product of H.C.F. and the quotient when it is divided by H.C.F.
- use the distributive property of multiplication over addition, to express the given expression as a product of H.C.F. and the quotient of the given expression divided by H.C.F.

Example 23: Find the H.C.F. of the terms and factorise the expression, $18x^3y^2 + 36xy^4 - 24x^2y^3$

Solution: H.C.F. of $18x^3y^2$, $36xy^4$ and $-24x^2y^3$ is $6xy^2$

$\therefore \quad 18x^3y^2 + 36xy^4 - 24x^2y^3 = 6xy^2\,(3x^2 + 6y^2 - 4xy)$

Worksheet 5

1. **Express the following as a product of its any two factors (in four different ways).**

   (i) $12x^2y$ (ii) $18ab^2$ (iii) $24c^2b$

2. **Find the H.C.F. of the following monomials.**

   (i) $2a^5$ and $12a^2$ (ii) $9x^3y$ and $18x^2y^3$ (iii) a^2b^3 and a^3b^2

   (iv) $15a^3$, $-45a^2$, $150a$ (v) $2x^3y^2$, $10x^2y^3$, $14xy$ (vi) x^3y^2, $-8y^2$

3. **Find the H.C.F. of the terms and factorise.**

(i) $5y - 15y^2$ (ii) $16m - 4m^2$ (iii) $8x^3y^2 + 8x^3$

(iv) $20x^3 - 40x^2 + 80x$ (v) $x^4y - 3x^2y^2 - 6xy^3$ (vi) $8x^2y^2 - 16xy^3 + 24xy$

FACTORISATION OF ALGEBRAIC EXPRESSION WHEN A BINOMIAL IS A COMMON FACTOR

To factorise an algebraic expression containing a binomial as a common factor, we write the expression as a product of the binomial and quotient obtained by dividing the given expression by its binomial. The following examples will illustrate the procedure:

Example 24: Factorise $(x + 2)y + (x + 2)x$

Solution: Here, $(x + 2)$ is a binomial common to both the terms of a given expression. So, we have,

$$(x + 2)y + (x + 2)x = (x + 2)\,(y + x)$$

$$= (x + 2)\,(y + x)$$

$\therefore$ $(x + 2)$ and $(y + x)$ are two factors of the given algebraic expression.

Example 25: Factorise $(5x^2 - 10xy) - 4x + 8y$

Solution: Taking $5x$ common from $(5x^2 - 10xy)$, we get

$$5x\,(x - 2y) - 4x + 8y$$

Now, taking -4 common from $-4x + 8y$, we get

$$5x\,(x - 2y) - 4\,(x - 2y)$$

$$= (x - 2y)\,(5x - 4)$$ [Taking $(x - 2y)$ common from both products]

Example 26: Factorise $(2x - 3y)\,(a + b) + (3x - 2y)\,(a + b)$

Solution: Here, in the given expression, $(a + b)$ is common to both the terms, so

$$= (a + b)\,(2x - 3y + 3x - 2y)$$

$$= (a + b)\,(5x - 5y)$$

Here, 5 is common to the term in $(5x - 5y)$, so we have

$$= (a + b)\,5\,(x - y) = 5\,(a + b)\,(x - y).$$

FACTORISATION BY REGROUPING THE TERMS

In the previous section, we have been able to factorise the given expression by taking a factor common from all the terms. But it is not always possible to have a common factor. So we try to form groups in such a way that a factor can be taken out from each group and the expression can be factorised. Let us take an example.

Example 27: Factorise $2ax + bx + 2ay + by$.

Solution: **I Method:** Forming a group of first two and the last two terms, we have

$$\begin{aligned} & (2ax + bx) + (2ay + by) \\ = \; & x(2a + b) + y(2a + b) \\ = \; & (2a + b)\,(x + y) \end{aligned}$$

Sometimes grouping can be done in many ways. In the above example, we can form the groups of $2ax$, $2ay$ and bx, by.

II Method:

$$\begin{aligned} & (2ax + 2ay) + (bx + by) \\ = \; & 2a(x + y) + b(x + y) \\ = \; & (2a + b)\,(x + y) \end{aligned}$$

We notice that in whatever way the grouping is done, we will always get the same factors.

Worksheet 6

Factorise the following expressions.

1. $(x + y)\,(2x + 3y) - (x + y)\,(x + 1)$
2. $9x\,(6x - 5y) - 12x^2(6x - 5y)$
3. $x^3(a - 2b) + x^2(a - 2b)$
4. $(a - b)^2 + (a - b)$
5. $3a\,(p - 2q) - b\,(p - 2q)$
6. $8\,(5x + 9y) + 12\,(5x + 9y)$
7. $1 + x + xy + x^2y$
8. $x^2 + xy + xz + yz$
9. $a(a + b) + 8a + 8b$
10. $a^2 + bc + ac + ab$
11. $a^2 + 2a + ab + 2b$
12. $ax + ay - bx - by$

VALUE BASED QUESTIONS

1. **On the occasion of *Van Mahotsav* it was decided by the Secretary of Residents Welfare Association to plant saplings of Ashoka trees and Mango trees in the locality. Accordingly Ashoka trees were planted in $(3x + 1)$ rows having $(x^3 + x^2 - 1)$ trees in each row and Mango trees were planted in $(x - 3)$ rows having $(2x^2 + 1)$ trees in each row.**

   (a) Find the total number of trees planted.

   (b) Discuss the importance of trees in our life.

2. **According to a data $(p^2 - 4p + 3)$ road accidents occurred in City A, $(6p - 7 + 2p^2)$ road accidents occurred in City B and $(p + 10 + 2p^2)$ road accidents occurred in City C.**

(a) Find the total number of road accidents occurred.

(b) What should be done to minimise accidents on roads?

BRAIN TEASERS

1. A. Tick (✓) the correct option.

(a) $(3p^2 - 14pq + 2r) - (14pq + 3p^2 + 2r^2)$ is a—

(i) monomial (ii) binomial (iii) trinomial (iv) none

(b) H.C.F. of the terms of the expression $(4p^3q^2r - 12pq^2r^2 + 16p^2q^2r^2)$ is–

(i) $4pq^2r$ (ii) $-4pq^2r$ (iii) $16p^3q^2r^2$ (iv) $-16p^3q^2r^2$

(c) For what value of 't' the expressions $(2x^2 - 5x + 10)$ and $(2x^2 - tx + 2t)$ are equal–

(i) 2 (ii) 5 (iii) 3 (iv) 4

(d) Value of 'p' if the expression $z^2 + 3z - p$ equals to 3 for $z = 2$ is–

(i) 6 (ii) 5 (iii) 7 (iv) 4

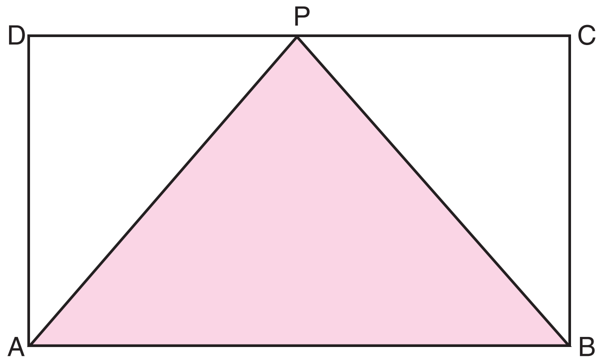
(e) Factors of $2x^3 + 5x - 6x^2 - 15$ are–

(i) $(2x^2 + 5)$ and $(x + 3)$ (ii) $(2x^2 - 5)$ and $(x + 3)$

(iii) $(2x^2 + 5)$ and $(x - 3)$ (iv) $(2x^2 - 5)$ and $(x + 3)$

B. Answer the following questions.

(a) In the given figure, ABCD is a rectangle with length $(p^2q - 5)$ and breadth $(q^2 + 4p)$. Find the area of shaded triangle PAB.



(b) By how much does the expression $72z^2 - 45z + 4$ exceed the expression $30z - 42z^2 - 17$?

(c) The perimeter of a triangle is $(x^2y + 10)$ units. One of the sides is of length $(x^2y - 4)$ units and another side is $(3 - 2x^2y)$ units. Find the length of the third side.

(d) Find the HCF of the terms of the expression

$$2b^3c + 4b^4c^2 + 16b^2c^2$$

Also, write the factors of the above expression.

(e) If area of a rectangle with length $(a - b)$ and breadth $(2a + b)$ is the same as the algebraic expression $Kab + 2a^2 - b^2$, find K.

2. Find the following products.

(i) $(13ax - 4)(5ay + 1)$

(ii) $(3x^2 + 5x - 7)(x + 5y)$

(iii) $-6a^2b(a^4 + b^4 - 3a^2b^2)$

(iv) $\left(\frac{3}{5}x - \frac{2}{9}y\right)(15x - 9y)$

(v) $(x^9)(-x^{10})(x^{11})(-x^{12})$

(vi) $0.9p^3q^3(10p - 20q)$

(vii) $(7x^2 - 11x + 10)(x^3 - x^2)$

(viii) $\left(\frac{10}{9}a^5b^6c^6\right)\left(-\frac{3}{2}b^2c\right)\left(\frac{6}{5}c^3d^4\right)\left(-a^3d^5\right)$

(ix) $(0.7x^3 - 0.5y^3)(0.7x^3 + 0.5y^3)$

(x) $(64a^2 - 56ab + 49b^2)(8a + 7b)$

3. Find the variable part in the product of:

(i) $127x^3y^9$, $255x^6y^5$ and $313y^2z^4$

(ii) $\frac{61}{17}a^2b^4$, $\frac{97}{43}b^3$, $\frac{29}{41}a^6b$ and $\frac{111}{123}$.

4. Express $\left(5x^6\right)\left(12x^2y\right)\left(\frac{3}{20}xy^2\right)$ as a monomial and then evaluate it for $x = 1$, $y = 2$.

5. Express $1.5a^2(10ab - 4b^2)$ as a binomial and then evaluate it for $a = -2$, $b = 3$.

6. Simplify and then verify the result for the given values.

(i) $(3x - 4y)(4x^2y + 3xy^2)$; $x = 2$, $y = -1$

(ii) $\left(\frac{1}{4}a^2 + \frac{5}{9}b^2\right)(a + b + ab)$; $a = 2$, $b = 3$

(iii) $(x^3y - y^2)(x^3y + y^2)$; $x = -1$, $y = -2$

(iv) $(2p + 3q)(4p^2 + 12pq + 9q^2)$; $p = \frac{1}{2}$, $q = \frac{1}{3}$

(v) $(m^2 + mn + n^2)(m - n)$; $m = 4$, $n = 3$

7. Simplify:

(i) $3x^2(3y^2 + 2) - x(x - 2xy^2) + y(2x^2y - 2y)$

(ii) $(2x + 7)(5x + 9) - (4x^2 + 1)(x - 3)$

(iii) $(y^2 - 7y + 4)(3y^2 - 2) + (y + 1)(y^2 + 2y)$

8. **Find the H.C.F. of the terms in the expression $3a^2b^2 + 6ab^2c^2 + 12a^2b^2c^2$.**
9. **Factorise:**

   (i) $ab^2 - bc^2 - ab + c^2$ (ii) $4(p + q)(3a - b) + 6(p + q)(2b - 3a)$

   (iii) $axy + bcxy - az - bcz$

ENRICHMENT QUESTION

Sum of first n natural numbers is given by the expression $\left(\dfrac{n^2 + n}{2}\right)$.

Sum of first five natural numbers (i.e. 1 + 2 + 3 + 4 + 5) will be

$$\frac{5^2 + 5}{2} = \frac{30}{2} = 15$$

Sum of first seven natural numbers (i.e. 1 + 2 + 3 + 4 + 5 + 6 + 7) will be

$$\frac{7^2 + 7}{2} = \frac{56}{2} = 28$$

Now, write down the sum of first ten natural numbers.

YOU MUST KNOW

1. An algebraic expression is a combination of constants and variables connected by +, –, × and ÷.
2. Algebraic expression with one term is called a monomial, with two terms, a binomial and with three terms, a trinomial.
3. Product of variables follows the rule of exponents: $x^m \times x^n = x^{m+n}$.
4. Product of monomials is the product of constants of the given monomials and their variables.
5. To multiply a monomial and a binomial, the monomial is multiplied with each term of the binomial and the products are added.
6. To multiply two binomials, each term of one binomial is multiplied with each term of the second binomial and the products are then added.
7. To multiply a binomial and a trinomial, each term of the binomial is multiplied with each term of the trinomial and the products are then added.

CHAPTER 7

LINEAR EQUATIONS IN ONE VARIABLE

INTRODUCTION

In Class-VI, you have learnt that an **equation** is a statement of equality involving one or more variables. For example,

$\boxed{2x + 3 = 0}$ $\boxed{12x + 4x^2 = 3}$ $\boxed{4y^2 - 9 = 0}$

$\boxed{2y^2 - 5 = 0}$ $\boxed{-5z + 15 = 5}$

All the above equations are in one variable as these involve equality sign and one variable. Recall that a **linear equation** in one variable is an equation in which the highest degree of the variable is one. Here, $2x + 3 = 0$ and $-5z + 15 = 5$ are linear equations in one variable.

SOLUTION OF LINEAR EQUATION IN ONE VARIABLE

Do you remember that any value of the variable which satisfies the given equation is called the **solution** or the **root** of the equation. Let us recall the rules for solving linear equations.

- Add the same quantity to both sides of an equation.
- Subtract the same quantity from both sides of an equation.
- Multiply both sides of an equation by the same non-zero number.
- Divide both sides of an equation by the same non-zero number.

It should be noted that when any of the above operations are applied, there is no change in the equation.

Remember

Any term of an equation may be taken to the other side by changing its sign without affecting the equality. This process is called **transposition.** So, by transposing a term, we simply change its sign and carry it to the other side of the equation. In other words. '+' sign of the term changes to '–' sign on the other side, 'x' sign of the factor changes to '÷' sign on the other side and vice-versa.

Let us take some examples to understand all these concepts once again.

Example 1: Solve $\frac{y}{5}+1=\frac{1}{15}$ and check the answer.

Solution:

$$\frac{y}{5}+1-1 = \frac{1}{15}-1 \qquad \text{(subtracting 1 from both sides)}$$

$$\frac{y}{5} = \frac{-14}{15}$$

$$\frac{y}{5}\times 5 = \frac{-14}{15}\times 5 \qquad \text{(multiplying both sides by 5)}$$

$$y = \frac{-14}{3}$$

Check: For $y = \frac{-14}{3}$

$$\text{L.H.S.} = \frac{-14}{15}+1=\frac{1}{15}$$

$$\text{R.H.S.} = \frac{1}{15}$$

So, L.H.S. = R.H.S.

Hence, $y=\frac{-14}{3}$ is a solution or root of the given equation.

Example 2: Solve $\frac{y}{2}+\frac{8}{3}=\frac{31}{6}$ and check the answer.

Solution:

$$\frac{y}{2}+\frac{8}{3} = \frac{31}{6}$$

$$\frac{y}{2}+\frac{8}{3}-\frac{8}{3} = \frac{31}{6}-\frac{8}{3} \qquad \text{(subtracting } \tfrac{8}{3} \text{ from both sides)}$$

$$\frac{y}{2} = \frac{31-16}{6}$$

$$\frac{y}{2} = \frac{15}{6}$$

$$y = \frac{15}{6}\times 2 \qquad \text{(multiplying both sides by 2)}$$

$$y = \frac{15}{3} = 5$$

$$y = 5$$

Check: By substituting the value of y in the given equation, we get

$$\text{L.H.S.} = \frac{5}{2} + \frac{8}{3} = \frac{15+16}{6} = \frac{31}{6}$$

$$\text{R.H.S.} = \frac{31}{6}$$

So, L.H.S. = R.H.S.

Therefore, $y = 5$ is a solution of the given equation.

Example 3: Solve $\frac{x}{2} - \frac{x}{3} = 8$ and check the answer.

Solution: L.C.M. of both the denominators 2 and 3 is 6. Multiplying both sides by 6, we get

$$6\left[\frac{x}{2} - \frac{x}{3}\right] = 48$$

$$\frac{6x}{2} - \frac{6x}{3} = 48$$

$$3x - 2x = 48$$

$$x = 48$$

Check: Putting $x = 48$ in L.H.S. of the given equation, we get

$$\text{L.H.S.} \quad \frac{48}{2} - \frac{48}{3} = 24 - 16 = 8 = \text{R.H.S.}$$

So, $x = 48$ is the solution of the given equation.

Example 4: Solve the equation:

$$2(x - 2) - 3(x - 3) = 5(x - 5) + 4(x - 8)$$

Solution: Removing the brackets from both sides of the equation, we get

$$2x - 4 - 3x + 9 = 5x - 25 + 4x - 32$$

Collecting the like terms on both sides of the equation and simplifying, we have

$$-x + 5 = 9x - 57$$

Transposing 5 to R.H.S. and $9x$ to L.H.S., the equation reduces to

$$-x - 9x = -57 - 5$$

$$-10x = -62$$

$$x = \frac{62}{10} = \frac{31}{5}$$

Thus, $x = \frac{31}{5}$ is a solution of the given equation. You can verify the answer by substituting the value of x in the given equation.

Example 5: Solve $\frac{6p+1}{3} + 1 = \frac{7p-3}{2}$ and check the answer.

Solution: Simplifying the expression on L.H.S. of the equation by taking L.C.M., we get

$$\frac{6p+1+3}{3} = \frac{7p-3}{2}$$

$$\frac{6p+4}{3} = \frac{7p-3}{2}$$

By cross multiplying, we get

$$2(6p + 4) = 3(7p - 3)$$

$$12p + 8 = 21p - 9$$

$$8 + 9 = 21p - 12p$$

[Transposing 9 to L.H.S. and 12p to R.H.S.]

$$17 = 9p$$

$$\frac{17}{9} = p$$

Check: For $p = \frac{17}{9}$, L.H.S. reduces to $\frac{6\left(\frac{17}{9}\right)+1}{3} + 1$.

$$\frac{\frac{34}{3}+1}{3} + 1 = \frac{37}{9} + 1 = \frac{37+9}{9} = \frac{46}{9}$$

$$\text{R.H.S.} = \frac{7\left(\frac{17}{9}\right)-3}{2} = \frac{\frac{119}{9}-3}{2} = \frac{119-27}{9\times 2} = \frac{92}{9\times 2} = \frac{46}{9}$$

So, L.H.S. = R.H.S.

Therefore, $p = \frac{17}{9}$ is a solution of the given equation.

Example 6: Solve $2.4x + 1.35 - 0.04x = 3.71x + 13.5$

Solution: $2.4x + 1.35 - 0.04x = 3.71x + 13.5$

By transposition, we have

$$2.4x - 0.04x - 3.71x = 13.5 - 1.35$$

$$2.4x - 3.75x = 12.15$$

$$-1.35x = 12.15$$

$$x = \frac{12.15}{1.35} = -9$$

So, $x = -9$ is a solution of the given equation.

You may verify that $x = -9$ satisfies the given equation.

Example 7: Solve the equation $ax + b = 0$, where a and b are rationals and $a \neq 0$.

Solution: We have $ax + b = 0$, where a and b are rationals and $a \neq 0$.

Transposing b to R.H.S., we get

$$ax = -b$$

Since, $a \neq 0$, we divide both sides by 'a'

$$\frac{ax}{a} = -\frac{b}{a}$$

$$\therefore \quad x = -\frac{b}{a}$$

So, $x = -\frac{b}{a}$ is the solution of the given equation.

Note:

In all the examples mentioned above, the linear equations to be solved are reduced to the form $ax + b = 0$ to find the solution. So, the general solution of an equation $ax + b = 0$

where a, b are rationals and $a \neq 0$ is given by $x = -\frac{b}{a}$

Worksheet 1

1. Solve the following equations and check your answers.

1. $5x - 2 = 18$
2. $\frac{1}{4}y + \frac{1}{2} = 5$
3. $\frac{3}{5}x - 6 = 3$
4. $3x + \frac{1}{5} = 2 - x$
5. $8x + 5 = 6x - 5$
6. $9z - 13 = 11z + 27$

7. $\frac{7}{y} + 1 = 29$

8. $\frac{3}{5}x + \frac{2}{5} = 1$

9. $4y - 2 = \frac{1}{5}$

10. $\frac{x}{2} + \frac{x}{4} = 12$

11. $\frac{2}{5}z = \frac{3}{8}z + \frac{7}{20}$

12. $\frac{2}{5}y - \frac{5}{8}y = \frac{5}{12}$

13. $3x + 2(x + 2) = 20 - (2x - 5)$

14. $13(y - 4) - 3(y - 9) = 5(y + 4)$

15. $(2z - 7) - 3(3z + 8) = 4z - 9$

16. $4(2y - 3) + 5(3y - 4) = 14$

17. $\frac{x}{2} - \frac{x}{3} = \frac{x}{4} + \frac{1}{2}$

18. $z - \frac{2z}{3} + \frac{z}{2} = 5$

19. $\frac{6y+1}{2} + 1 = \frac{7y-3}{3}$

20. $\frac{6x-2}{5} = \frac{2x-1}{3} - \frac{1}{3}$

21. $\frac{z-1}{3} = 1 + \frac{z-2}{4}$

22. $2x - 3 = \frac{3}{10}(5x - 12)$

23. $3(y - 3) = 5(2y + 1)$

24. $0.6x + 0.8 = .28x + 1.16$

25. $2.4(3 - x) - 0.6(2x - 3) = 0$

APPLICATION OF LINEAR EQUATIONS (WORD PROBLEMS)

In this section, you will study the formulation and solution of some practical problems stated in words. These problems involve the relation among the unknown and known quantities which are expressed as mathematical expressions and equations. You have already learnt in Class-VI to convert the given problems in the form of expressions and equations. We will now extend this concept to find the solution of a given word problem. Finding a solution of a word problem involves three steps:

Step I. Formation of an equation.

Step II. Solving an equation.

Step III. Interpreting the solution.

The following steps should be taken while you solve any word problem:

- Read the given problem very carefully and note what is given and what is required.
- Write the unknown quantities by any letter, such as x, y, z, u, v, w, etc.
- Translate the statement of the given word problem step by step (or word by word) into mathematical statement.
- Using the given conditions in the problem, formulate an equation.
- Solve the equation for the unknown.

After finding the solution, you can check whether the given solution satisfies the equation or not.

The following examples will illustrate these steps:

Example 8: When 5 is added to three times a number, we obtain 44. Find the number.

Solution: Let the number be x.

According to question;

$$3x + 5 = 44$$

$$3x = 39 \qquad \text{(Transposing 5 to R.H.S.)}$$

$$x = 13$$

So the number $= 13$.

Check:

For $x = 13$, then $3 \times 13 + 5 = 44$ which is true.

Example 9: The sum of two numbers is 95. If one exceeds the other by 3, find the numbers.

Solution: Let one number be y.

Other number $= y + 3$

According to question,

$$y + (y + 3) = 95$$

$$2y + 3 = 95$$

$$2y = 95 - 3 \qquad \text{(Transposing 3 to R.H.S.)}$$

$$2y = 92$$

$$y = 46$$

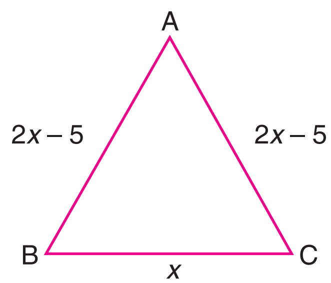
So the number is $= 46$

$\therefore$ Other number $= 46 + 3 = 49$

Check: $46 + 49 = 95$, i.e. the sum of two numbers is 95.

Example 10: Two equal sides of a triangle are each 5 m less than twice the third side. If the perimeter of the triangle is 55 m, find the lengths of its sides.

Solution: Let the third side of the triangle be x metres. Then the two equal sides will be $(2x - 5)$ m each. According to the problem,



$$\begin{aligned}
\text{Perimeter of } \Delta ABC &= AB + BC + CA = 55 \text{ m} \\
\therefore \quad 2x - 5 + x + 2x - 5 &= 55 \\
5x - 10 &= 55 \\
5x &= 65 \\
x &= 13 \\
\text{So, third side} &= 13 \text{ m} \\
\text{Other two sides} &= 2 \times 13 - 5 = 21 \text{ m}
\end{aligned}$$

$\therefore$ The sides are 13 m, 21 m and 21 m

Check: 13 + 21 + 21 = 55 m is the perimeter of ΔABC.

Example 11: A piggy bank contains ₹ 370 in the notes of denominations of 10 and 50. If the number of 10 rupee notes is one more than that of 50 rupee notes, find the number of notes of each type.

Solution: Let us suppose the number of 50 rupee notes = x.

Then the number of 10 rupee note = $(x + 1)$

Value of 50 rupee notes = $50 \times x$

Value of 10 rupee notes = $10(x + 1)$

Total money = $50 \times x + 10(x + 1) = 50x + 10x + 10$

According to the problem,

$$\begin{aligned}
50x + 10x + 10 &= 370 \\
60x &= 360 \\
x &= \frac{360}{60} = 6
\end{aligned}$$

$\therefore$ Number of 50 rupee notes = 6

Number of 10 rupee notes = 7

Check: Total money = $50 \times 6 + 10 \times 7$ = ₹ 370.

Example 12: Present age of Veena's mother is four times Veena's age. Five years hence, her age will be 21 years more than Veena's age. Find their present ages.

Solution: Let present age of Veena = x yrs

Veena's mother's age = $4x$ yrs

After five years,

Veena's age = $(x + 5)$ yrs

Mother's age = $(4x + 5)$ yrs

According to question,

$$\begin{aligned} 4x + 5 &= x + 5 + 21 \\ 3x &= 21 \\ x &= 7 \text{ yrs} \\ \therefore \quad \text{Veena's age} &= 7 \text{ yrs} \\ \text{Mother's age} &= 7 \times 4 = 28 \text{ yrs} \end{aligned}$$

Worksheet 2

1. **Adding 4 to twice a number yields $\frac{25}{6}$. Find the number.**
2. **A number when added to its two-thirds is equal to 55. Find the number.**
3. **A number when multiplied by 4 exceeds itself by 45. Find the number.**
4. **Find a number which when multiplied by 8 and then reduced by 9 is equal to 47.**
5. **The sum of two numbers is 72. If one of the number is 6 more than the other, find the numbers.**
6. **The sum of two number is 99. If one exceeds the other by 9, find the numbers.**
7. **One number is 10 more than the other. If their sum is 52, find the numbers.**
8. **The length of a rectangle is 20 cm more than its breadth. If the perimeter is 100 cm, find the dimension of the rectangle.**
9. **The length of a rectangle is three times its width. It the perimeter is 84 m, find the length of the rectangle.**
10. **Two equal sides of an isoscles triangle are each 2 cm more than thrice the third side. If the perimeter of triangle is 67 cm, find the lengths of its sides.**
11. **Length of a rectangle is 16 cm less than twice its breadth. If the perimeter of the rectangle is 100 cm, find its length and breadth.**
12. **Find two consecutive positive integers whose sum is 63.**
13. **A sum of ₹ 800 is in the form of denominations of ₹ 10 and ₹ 20. If the total number of notes is 50, find the number of notes of each type.**
14. **In a class of 49 students, number of girls is $\frac{2}{5}$ of the boys. Find the number of boys in the class.**

15. **Leena has 117 rupees in the form of 5 rupee coins and 2 rupee coins. The number of 2 rupee coins is 4 times that of 5 rupee coins. Find the number of coins of each denomination.**

16. **A total of ₹ 80,000 is to be distributed among 200 persons as prizes. A prize is either of ₹ 500 or ₹ 100. Find the number of each prize.**

17. **When $\frac{1}{3}$ is subtracted from a number and the difference is multiplied by 4, the result is 28. Find the number.**

18. **Sudesh is twice as old as Seema. If six years is subtracted from Seema's age and four years are added to Sudesh's age, Sudesh will be four times Seema's age. How old were they three years ago?**

19. **The ages of Leena and Heena are in the ratio 7 : 5. Ten years hence, the ratio of their ages will be 9 : 7. Find their present ages.**

20. **Vikas is three years older than Deepika. Six years ago, Vikas's age was four times Deepika's age. Find the ages of Deepika and Vikas.**

VALUE BASED QUESTIONS

1. **Aman is an avid reader. He has a good collection of books. In his summer vacation he read a book of 300 pages in four days.**

   (a) If the number of pages he read on second, third and fourth day is half the number of pages he read on previous day, find the number of pages he read on first day.

   (b) Discuss the importance of reading books.

2. **Students of Rotary Club prepared 'Tie and Dye *Dupattas*', 'Wall Hangings' and 'Hand Bags' for Sale in an Exhibition to help the poor people. They earned a total profit of ₹ 2000.**

   (a) If profit on *Dupattas* is ₹ 30 less than two times the profit on Wall Hangings and profit on Hand Bags is ₹ 80 more than twice the profit on Wall Hangings. Find the profit on Wall Hangings.

   (b) What value is exhibited by the students?

BRAIN TEASERS

1. A. Tick (✓) the correct option.

(a) $x + x + x + x =$ ________

(i) x^4 (ii) $4x$ (iii) x (iv) $4x^4$

(b) $(9z + 7) - 5(2z - 3) + 3(2z + 3) =$ ________

(i) $5z + 31$ (ii) $-5z + 31$ (iii) $5z - 31$ (iv) $-5z - 31$

(c) If $4x - 3 = 21$, what is the value of $(3x - 5)$?

(i) 16 (ii) 14 (iii) 13 (iv) 15

(d) Which of the following is a solution of the equation–

$$3x - 7 = 7 - 4x$$

(i) $x = 0$ (ii) $x = 14$ (iii) $x = 2$ (iv) $x = 1$

(e) $x = 5$ is a solution of the equation–

(i) $\frac{3}{5}x - 6 = 3$ (ii) $2x + 2 = 4x$

(iii) $4x - 2 = 3x + 3$ (iv) $3(x - 3) = 5(2x + 1)$

B. Answer the following questions.

(a) A confectioner had some boxes for burger and he bought 50 more boxes. After two days half of these boxes were used and only 40 boxes were left. Find the number of boxes he had in the beginning.

(b) Find K, so that $x = 3$ is a solution of the equation $Kx + 8 = x - 7$.

(c) Seema had a wire of some length. After making a square of side 4 cm with it, 5 cm wire left. Find the length of wire Seema had.

(d) If A = $(2a - 3)$ and B = $(5 - 3a)$ be two algebraic expressions such that $2A + B = 6$, then find the value of a.

(e) Check whether $y = 1$ is a solution of the equation–

$1.2(3y - 4) - 0.3(2y - 6) = 0$ or not.

2. Solve the following equations.

(i) $\frac{x}{2} + \frac{x}{3} - \frac{x}{4} = 7$ (ii) $\frac{2}{3}(y - 5) - \frac{1}{4}(y - 2) = \frac{9}{2}$

(iii) $13(z - 4) - 3(z - 9) - 5(z + 4) = 0$

(iv) $(x + 2)(x + 3) + (x - 3)(x - 2) - 2x(x + 1) = 0$

(v) $\dfrac{2x+14}{3x+6} = 4$

(vi) $a - \left(\dfrac{a-1}{2}\right) = 1 - \dfrac{(a-2)}{3}$

(vii) $\dfrac{x+2}{6} - \left[\dfrac{11-x}{3} - \dfrac{1}{4}\right] = \dfrac{3x-14}{12}$

(viii) $\dfrac{2}{3x} - 1 = \dfrac{1}{12}$

3. The numerator of a fraction is 6 less than the denominator. If 3 is added to the numerator, the fraction becomes equal to $\frac{2}{3}$. Find the original fraction.

4. A man travelled $\frac{2}{5}$th of his journey by train, $\frac{1}{3}$rd by taxi, $\frac{1}{6}$th by bus and remaining 10 km on foot. Find the length of his total journey.

5. A table costs ₹ 200 more than a chair. The price of two tables and three chairs is ₹ 1400. Find the cost of each.

6. Anu is four years older than Sunil. Eight years ago, Anu was three times Sunil's age. Find the ages of Sunil and Anu.

YOU MUST KNOW

1. The standard form of linear equation in one variable x is $ax + b = 0$, where a and b are rationals and $a \neq 0$.
2. While solving a linear equation—
   (a) add and subtract the same quantity on both sides of an equation.
   (b) multiply both sides of an equation by the same non-zero number.
   (c) divide both sides of an equation by the same non-zero number.
3. Transposing a term means simply changing its sign and taking it to the other side of the equation.

   $+$ changes to $-$ and vice versa.

   $\times$ changes to $\div$ and vice versa.
4. Finding a solution to a word problem involves three steps–
   (a) Forming an equation.
   (b) Solving an equation.
   (c) Interpreting the solution.